

# Extension of the Eddy Dissipation Concept for turbulence/chemistry interactions to MILD combustion

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## Abstract

Over the past 30 years, the Eddy Dissipation Concept (EDC) has been widely applied in the industry for the numerical simulations of turbulent combustion problems. The success of the EDC is mainly due to its ability to incorporate detailed chemical mechanisms at an affordable computational cost compared to some other models. Detailed kinetic schemes are necessary in order to capture turbulent flames where there is strong coupling between the turbulence and chemical kinetics. Such flames are found in Moderate and Intense Low-oxygen Dilution (MILD) combustion, where chemical time scales are increased compared with conventional combustion, mainly because of slower reactions (due to the dilution of reactants). Recent modelling studies have highlighted limitations of the standard EDC model when applied to the simulation of MILD systems, noticeably a significant overestimation of temperature levels. Modifications of the model coefficients were proposed to account for the specific features of MILD combustion, i.e. an extension of the reaction region and the reduction of maximum temperatures. The purpose of the present paper is to provide functional expressions showing the dependency of the EDC coefficients on dimensionless flow parameters such as the Reynolds and Damköhler numbers, taking into account the specific features of the MILD combustion regime, where the presence of hot diluent and its influence on the flow and mixing fields impacts on the reaction rate and thermal field. The approach is validated using detailed experimental data from flames stabilized on the Adelaide Jet in Hot Co-flow (JHC) burner at different co-flow compositions (3%, 6% and 9% O<sub>2</sub> mass fraction) and fuel-jet Reynolds numbers (5,000, 10,000 and 20,000). Results show promising improvement with respect to the standard EDC formulation, especially at diluted conditions and medium to low Reynolds numbers.

**Keywords:** Eddy Dissipation Concept; Energy cascade model; Flameless combustion; MILD combustion; Turbulence/chemistry interactions.

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## 13 1. Introduction

14 New breakthroughs in clean energy are needed to provide our society with the necessary resources in a  
15 way that also protects the environment and addresses the climate change issue. The need for innovation  
16 is particularly important in combustion, considering that the energy derived from burning fossil fuels (coal,  
17 petroleum or natural gas) supplies over two thirds of the total world energy needs. A certain number of  
18 new combustion technologies have been proposed in recent years. Among them, Moderate or Intense Low-  
19 oxygen Dilution (MILD) [1–3] combustion is certainly one of the most promising, as it is able to provide  
20 high combustion efficiency with low pollutant emissions. This mode of combustion is achieved through the  
21 strong exhaust gas and heat recirculation, achieved by means of the internal aerodynamics of the combustion  
22 chamber in conjunction with high-velocity burners [1]. Heat recovery by preheating the oxidant stream can  
23 also help in improving thermal efficiency and maintaining the MILD regime. The resulting combustion regime  
24 features reduced local oxygen levels, distribution of reaction over the whole combustion chamber, no visible  
25 or audible flame and thus the name flameless. The temperature field is more uniform due to absence of  
26 temperature peaks, which drastically reduces NO<sub>x</sub> formation [1, 2, 4–6], while ensuring complete combustion  
27 and low CO emissions [7–10]. MILD combustion can accommodate large fuel flexibility, representing an ideal  
28 technology for low-calorific value fuels [11–14], high-calorific industrial wastes [15] as well as for hydrogen-  
29 based fuels [16, 17].

30 In recent years, attention has been paid to MILD combustion modelling, due to the very strong tur-  
31 bulence/chemistry interactions of such a combustion regime. The Damköhler number in MILD conditions  
32 usually approaches unity [17] and both mixing and chemistry need to be taken into account with appropriate  
33 turbulent combustion models. This has also been proven by Parente et al. [18], who analysed the correla-  
34 tion structure of MILD combustion data [19] using Principal Component Analysis (PCA) and showed that  
35 the standard flamelet approach is not suited for such combustion regime. Recently, successful predictions  
36 of different MILD combustion cases have been reported [17, 20–22] using Reynolds-Averaged Navier-Stokes  
37 (RANS) modelling and the Eddy Dissipation Concept (EDC) [23]. However, several studies of the Jet in  
38 Hot Co-flow (JHC) configuration [19] also reported that the standard EDC tends to over-predict maximum  
39 temperatures when applied to the MILD combustion regime [24, 25]. Recently, De et al. [26] carried out  
40 a detailed study on the performance of the EDC model on the Delft Jet in Hot Co-flow burner (DJHC)  
41 emulating MILD conditions. The authors showed that the model described correctly the mean velocity pro-  
42 files and the Reynolds shear stress distributions, but showed significant discrepancies between measured and  
43 predicted temperatures. The mean temperature field showed systematic deviations from experimental data,  
44 due to the under-prediction of the lift-off height and the over-estimation of the maximum temperature level.  
45 This is mainly due to the over-estimations of the mean reaction rate in the EDC model. The authors showed  
46 that the prediction could be improved by adapting the standard coefficients of the classic EDC model, in  
47 particular increasing the time scale value,  $C_\tau$ , from 0.4083 to 3. The results were further confirmed for the  
48 analysis of the Adelaide JHC flames with methane/hydrogen mixtures [27] and with several ethylene-based

blends [28]. Recently, Evans et al. [29] showed that adjusting the EDC coefficients  $C_\tau$  and  $C_\gamma$  from their default value, 0.4082 and 2.1377, to 3.0 and 1.0, respectively, results in significantly improved performance of the EDC model under MILD conditions. Although the modification of the coefficients was shown to provide improved agreement between experiments and numerical simulations, it is still necessary to identify clear guidelines for the modification of the model coefficients in the context of MILD combustion, based on the specific turbulence and chemical features of such a regime. Shiehnjadhesar et al. [30] showed, for instance, that the standard EDC is not applicable for turbulent Reynolds values below 64 and proposed a hybrid Eddy Dissipation Concept/Finite-rate model calculating an effective reaction rate weighting a laminar finite-rate and a turbulent reaction rate, depending on the local turbulent Reynolds number of the flow.

The purpose of the present paper is to provide functional expressions showing the dependency of the EDC coefficients on dimensionless flow parameters such as the Reynolds and the Damköhler numbers. After a brief description of EDC and of the energy cascade model it relies on, the novel approach for the determination of the EDC coefficients will be presented. Results for the Adelaide JHC at different co-flow composition (3%, 6% et 9%  $O_2$  mass fraction) and fuel-jet Reynolds numbers (5000, 10000 and 20000) will be presented, to assess the soundness of the current approach.

## 2. Eddy Dissipation Concept

The Eddy Dissipation Concept (EDC) by Magnussen [23] for turbulent combustion has found wide application for the simulation of turbulent reacting flows, especially for cases where combustion kinetics plays a major role, as it happens for MILD conditions. EDC has the advantage of incorporating detailed kinetics at a computational cost which is affordable when compared to more sophisticated models such as the transported PDF methods. This advantage is maximised when EDC is used in conjunction with *in-situ* adaptive tabulation (ISAT) [31].

According to the EDC model, combustion occurs in the regions of the flow where the dissipation of turbulence kinetic energy takes place. Such regions are denoted as fine structures and they can be described as perfectly stirred reactors (PSR). The mass fraction of the fine structures,  $\gamma_\lambda$ , and the mean residence time of the fluid within them,  $\tau^*$ , are provided by an energy cascade model [32], which describes the energy dissipation process as a function of the characteristic scales:

$$\gamma_\lambda = \left( \frac{3C_{D2}}{4C_{D1}^2} \right)^{\frac{1}{4}} \left( \frac{\nu\epsilon}{k^2} \right)^{\frac{1}{4}} = C_\gamma \left( \frac{\nu\epsilon}{k^2} \right)^{\frac{1}{4}} \quad (1)$$

and

$$\tau^* = \left( \frac{C_{D2}}{3} \right)^{\frac{1}{2}} \left( \frac{\nu}{\epsilon} \right)^{\frac{1}{2}} = C_\tau \left( \frac{\nu}{\epsilon} \right)^{\frac{1}{2}} \quad (2)$$

where  $\nu$  is the kinematic viscosity and  $\epsilon$  is the dissipation rate of turbulent kinetic energy,  $k$ .  $C_{D1}$  and  $C_{D2}$  are model constants set equal to 0.135 and 0.5, respectively, leading to fine structure volume and residence

79 time constants equal to  $C_\gamma = 2.1377$  and  $C_\tau = 0.4083$ . Fine structures are assumed to be isobaric, adiabatic  
80 perfectly stirred reactors. The mean (mass-based) source term in the conservation equation for the  $i^{th}$  species  
81 is modelled as suggested by Gran and Magnussen [33]:

$$\bar{\omega}_i = -\frac{\bar{p}\gamma_\lambda^2}{\tau^*(1-\gamma_\lambda^3)}(\tilde{y}_i - y_i^*), \quad (3)$$

82 where  $\bar{p}$  denotes the mean density of the mixture,  $y_i^*$  is the mass fraction of the  $i^{th}$  species in the fine structures  
83 and  $\tilde{y}_i$  represents the mean mass fraction of the  $i^{th}$  species between the fine structures and the surrounding  
84 state (indicated as  $y_i^0$ ):

$$\tilde{y}_i = \gamma_\lambda^3 y_i^* + (1 - \gamma_\lambda^3) y_i^0. \quad (4)$$

85 As indicated above, the expressions for  $\gamma_\lambda$  and  $\tau^*$  used in the mean reaction rate for the  $i^{th}$  species are  
86 obtained from an energy cascade model, based on Kolmogorov's theory. In the following the model is briefly  
87 summarized, to highlight the main hypothesis behind it. Then, the proposed modification of the EDC  
88 standard coefficients will be presented and discussed.

### 89 2.1. Energy cascade model

90 The energy cascade model for EDC [32] starts with the transfer rate of mechanical energy,  $w'$ , from the  
91 mean flow to the large turbulent eddies. The sum of the heat generated at each level,  $\sum_i q_i$ , is assumed  
92 to be equal to the turbulent dissipation rate  $\epsilon$ . The first cascade level is characterized by a velocity scale  
93  $u' = \sqrt{2/3k}$  and a length scale  $L'$ , giving a strain rate  $\omega' = u'/L'$ , and it represents the whole turbulence  
94 spectrum because it contains the effect of smaller scales. In the energy cascade model, it is assumed that the  
95 strain doubles at each level, so that  $\omega'' = u''/L'' = 2\omega'$ . The strain rate at level  $n$  is  $\omega_n = 2\omega_{(n-1)}$ . In the  
96 original model formulation, the last level is described by scales  $\omega^*$ ,  $u^*$ ,  $L^*$ , which are considered to be of the  
97 same order of Kolmogorov scales,  $\omega_k$ ,  $u_k$ ,  $L_k$ .

98 The rate of production of mechanical energy,  $w_i$ , and the rate of viscous dissipation,  $q_i$ , at each level of  
99 the cascade are expressed [32] in analogy to the production and dissipation terms appearing in the equation  
100 of turbulent kinetic energy,  $k$ . This implies, for level  $n$ :

$$w_n = \frac{3}{2}C_{D1}\omega_n u_n^2 = \frac{3}{2}C_{D1}\frac{u_n^3}{L_n}, \quad (5)$$

101

$$q_n = C_{D2}\nu\omega_n^2 = C_{D2}\nu\frac{u_n^2}{L_n^2} \quad (6)$$

102 and, by the conservation of energy

$$w_n = q_n + w_{n+1}. \quad (7)$$

103 At the fine structure level, where reaction occurs, the energy is dissipated into heat,

$$w^* = \frac{3}{2}C_{D1}\frac{u^{*3}}{L^*} = q^* = C_{D2}\nu\frac{u^{*2}}{L^{*2}}. \quad (8)$$

104 In the original energy cascade formulation, the value of  $C_{D2}$  was selected as best fit for several types of flow,  
 105 whereas  $C_{D1}$  was chosen using the approximation that for  $Re \gg 1$  nearly no dissipation takes place at the  
 106 highest cascade level. This implies:

$$\varepsilon = w' = q' + w'' = w'' = \frac{3}{2} C_{D1} \frac{u'^3}{L}. \quad (9)$$

107 Under this assumption, a relation can be found between  $C_{D1}$  and the  $k - \varepsilon$  turbulent model constant  $C_\mu$ , *via*  
 108 the definition of the turbulent viscosity,  $\nu_T$ :

$$u' L' = \frac{3}{2} C_{D1} \frac{u'^4}{\varepsilon} = \frac{2}{3} C_{D1} \frac{k'^2}{\varepsilon}. \quad (10)$$

109 Considering the definition of  $\nu_T$  in the  $k - \varepsilon$  turbulent model,  $\nu_T = c_\mu \frac{k'^2}{\varepsilon}$ , we conclude that  $2/3 C_{D1}$  cor-  
 110 responds to the constant  $C_\mu = 0.09$ , which gives  $C_{D1} = 0.135$ . On the other hand, summing up all level  
 111 contributions to dissipation, and performing an energy balance on the last energy level, two additional rela-  
 112 tions are found [32]:

$$\varepsilon = \frac{4}{3} C_{D2} \nu \frac{u^{*2}}{L^{*2}} \quad (11)$$

113

$$\varepsilon = 2 C_{D1} \frac{u^{*3}}{L^*}. \quad (12)$$

114 Combining Equations (11) and (12), we conclude that the fine structure scale is of the same order of the  
 115 Kolmogorov one:

$$Re^* = \frac{u^* L^*}{\nu} = \frac{2 C_{D2}}{3 C_{D1}} = 2.5. \quad (13)$$

116 The classical cascade model described here has been developed for high Reynolds number flows, with clear  
 117 separation between turbulent scales. However, in MILD combustion, there is no longer a clear separation  
 118 between large and small scales of turbulence, and reaction can occur over a wide range of scales [34]. Therefore,  
 119 the chemical reactions proceed in a thick distributed reaction zone comparable to the integral length scale,  
 120 leading to a modification of the characteristic scales of the reaction structures, due to the transfer of energy to  
 121 higher frequencies than those of the reacting structures in the spectrum. It is therefore necessary to revise the  
 122 cascade model, to deal with the specific features of the MILD combustion regime, and clarify the dependency  
 123 of the energy cascade parameters on the flow and reaction structure characteristics, using the Reynolds and  
 124 Damköhler numbers.

## 125 2.2. Determination of energy cascade coefficients in MILD combustion

126 In MILD combustion, the dilution and preheating of the reactants generate a unique “distributed” reaction  
 127 zone [34]. The system evolves towards perfectly mixed conditions and the reaction process is characterized  
 128 by a Damköhler number approaching unity. As pointed out in the introduction, this has led several research  
 129 groups to modify the classic EDC model coefficients to achieve better predictions of experimental data. In

130 particular, the choice of the coefficients proposed by Evans et al. [29] has interesting implications for what  
 131 concerns the fine structure characteristics. Using the values of 3.0 and 1.0 for  $C_\tau$  and  $C_\gamma$ , respectively, in  
 132 Equation (13), a characteristic Reynolds number  $Re^* = 4$  is obtained, which indicates that the reacting  
 133 structures in MILD combustion have larger characteristic dimensions than in traditional combustion sys-  
 134 tems. This was recently confirmed by the analysis of Minamoto et al. [34], who pointed out that reacting  
 135 regions in MILD combustion are distributed over a good portion of the computational domain and the inter-  
 136 action between reaction zones leads to an appearance of distributed reaction, resulting in relatively uniform  
 137 temperature distribution.

138 We assume that the MILD combustion happens in the so-called Distributed Reaction Regime, to base  
 139 our revision of the standard cascade model. Such a combustion regime is associated mainly with small-scale,  
 140 high-intensity turbulence. In such a regime,  $u' \gg S_L$  and  $L' < \delta_L$ , meaning that Kolmogorov scales are  
 141 able to enter and to thicken the preheating zone and, possibly, the reaction region, leading to a thickened  
 142 and distributed flame structure [35, 36]. In such a scenario, it is appropriate to estimate the characteristic  
 143 speed of the reacting fine structures from the turbulent flame speed. The validity of such hypothesis requires  
 144 that a flame front can be still defined at the characteristic scales of the reacting structures, and this is indeed  
 145 the case for the regime under investigation, given its distributed nature determined by the high-intensity  
 146 turbulence. When dealing with this regime, it is a common practice to: i) model the effects of turbulence on  
 147 combustion as enhancement of heat and mass transport; ii) employ the classic expression by Damköhler for  
 148 turbulent flame speed,  $S_T$  [36, 37]:

$$S_T = S_L \sqrt{\frac{\alpha_T + \alpha}{\alpha}} \approx S_L \sqrt{\frac{\nu_T + \nu}{\nu}} = S_L \sqrt{Re_T + 1} \quad (14)$$

149 where  $Re_T = k^2/(\nu\epsilon)$  is the turbulent Reynolds number. The use of premixed quantities such as the laminar  
 150 flame speed is justified by the large degree of partial premixing occurring in MILD conditions. The very strong  
 151 recirculation determines a modification of the reaction region which evolves to perfectly stirred reactor (PSR)  
 152 conditions. The nature of reacting structures and the suitability of existing modelling paradigms in MILD  
 153 combustion has been recently investigated by Minamoto and Swaminathan [38]. They showed that MILD  
 154 reaction zones are highly-convoluted, contorted and pancake-like structures, spread over a large portion of the  
 155 computational domain resulting in a relatively broad reaction zone. By means of a systematic comparison  
 156 between numerical simulations and DNS data, the authors showed that the PSR modelling paradigm is  
 157 applicable in the (RANS and LES) modelling of MILD combustion, as also demonstrated in [39, 40]. Based  
 158 on this observation, the use of Equation (14) appears a good first-order estimate of the reacting structure  
 159 characteristic velocity for MILD conditions, and it can be used to infer the dependency of the energy cascade  
 160 model coefficients  $C_{D1}$  and  $C_{D2}$  on the dimensionless reacting flow numbers.

161 From Eq. (11), we know that  $\epsilon \propto C_{D2}\nu u^{*2}/L^{*2}$ . Considering that  $u^*$  is the characteristic speed of the  
 162 turbulent reacting fine structures, i.e.  $u^* \sim S_T = S_L \sqrt{Re_T + 1}$ , one gets from Eq. (14):

$$\epsilon \propto C_{D2} \nu \frac{u^{*2}}{L^{*2}} = C_{D2} \nu \frac{S_L^2 (Re_T + 1)}{L^{*2}}. \quad (15)$$

163 The length scale  $L^*$  can be interpreted as the characteristic linear dimension of the reacting fine structures,  
 164 being the reactions distributed over many turbulent length scales. This implies that the ratio  $L^*/S_L$  indicates  
 165 a characteristic chemical time scale,  $\tau_c$ , of the reacting structures, in line with the classic treatment of  
 166 turbulent premixed flames [36]. Consequently,  $\tau_c$  can be expressed as a function of the Kolmogorov mixing  
 167 time scale,  $\tau_\eta$ , using the flow Damköhler number. Thus, dissipation can be expressed as:

$$\epsilon \propto \frac{[C_{D2} \nu (Re_T + 1)]}{\tau_c^2} = \frac{[C_{D2} \nu (Re_T + 1) Da_\eta^2]}{\tau_\eta^2}. \quad (16)$$

168 Here, the pertinent mixing time scale for comparison is the Kolmogorov one, as indicated in the original energy  
 169 cascade model by Ertesvåg and Magnussen [32], explaining the use of the symbol  $Da_\eta$ , to indicate that the  
 170 Damköhler number is evaluated at the Kolmogorov scale  $\eta_k$ ,  $Da_\eta = \tau_\eta/\tau_c$ . Such a choice is motivated by the  
 171 need of comparing the reaction process occurring in the fine structures to the molecular mixing process at  
 172 the Kolmogorov scale. The problem could be also treated by introducing the Karlovitz ( $Ka$ ) number, which  
 173 intrinsically adopts the Kolmogorov scale, leading to an equivalent formulation. However, the interpretation  
 174 in terms of Damköhler number appears more immediate, being the observed  $Da_\eta$  number for MILD systems  
 175 of order unity [41]. Expressing  $\tau_\eta = (\nu/\epsilon)^{\frac{1}{2}}$ , we get the following dependency of  $C_{D2}$  on  $Re_T$  and  $Da_\eta$ :

$$C_{D2} \propto \frac{1}{[Da_\eta^2 (Re_T + 1)]}. \quad (17)$$

176 Given the definition of  $C_\tau$  (Eq. (2)), we obtain:

$$C_\tau \propto \frac{1}{Da_\eta \sqrt{Re_T + 1}} \quad (18)$$

177 Equation (18) provides a theoretical basis and confirms the recent findings in [26, 27]. In particular, it shows  
 178 that for combustion regimes characterized by low Damköhler numbers, the fine structure time coefficient  
 179 should be increased, to account for: i) the wider reaction regions; ii) the reduction of driving forces due to  
 180 the smoothed gradients; and iii) the reduction of temperature due to higher dilution. Moreover, Eq. (18) also  
 181 introduces an explicit dependence on the turbulent Reynolds number, indicating that for decreasing  $Re_T$  the  
 182 aforementioned phenomena become even more relevant.

183 A similar procedure can be carried out to determine the dependency of  $C_\gamma$  on  $Re_T$  and  $Da_\eta$ . From Eq.  
 184 (13) we can find a relationship between  $C_{D2}$  and  $C_{D1}$ ,  $C_{D2}/C_{D1} = 3/2 Re^* = \frac{3}{2} \frac{u^* L}{\nu}$ . Using Eq. (14) for  $u^*$ ,  
 185 we get

$$\frac{C_{D2}}{C_{D1}} = \frac{3}{2} S_L \sqrt{\frac{\nu_T + 1}{\nu^2}} L^*, \quad (19)$$

186 and, since  $S_L \propto \sqrt{\nu/\tau_c}$  [36], the ratio  $C_{D2}/C_{D1}$  can be expressed as

$$\frac{C_{D2}}{C_{D1}} = \frac{3}{2} \frac{L^*}{S_L} \frac{\sqrt{\frac{\nu_T}{\nu} + 1}}{\tau_c} \propto \sqrt{Re_T + 1}. \quad (20)$$

187 Based on the definition of  $C_\gamma$ (Eq. 1), the following expression is found:

$$C_\gamma = \left( \frac{3C_{D2}}{4C_{D1}^2} \right)^{\frac{1}{4}} \propto \left[ \frac{(Re_T + 1)}{C_{D2}} \right]^{\frac{1}{4}} \propto Da_\eta^{\frac{1}{2}} (Re_T + 1)^{\frac{1}{2}} \quad (21)$$

188 which indicates that the fine structure coefficient should be decreased for decreasing  $Re_T$  and  $Da_\eta$ . This  
 189 result can be interpreted considering that for low  $Re_T$  and  $Da_\eta$  the fine structures are more distributed and  
 190 their local mass fraction decreases. Moreover, it is important to note that the dependency on the Damköhler  
 191 number for such a coefficient is less important than for  $C_T$ .

### 192 3. Validation test cases

193 In order to validate the proposed approach, data are needed at different  $Re_T$  and  $Da_\eta$ . To this end, the  
 194 Adelaide Jet in Hot Co-flow burner [19] represents an ideal test-case for our purposes, thanks to the availability  
 195 of detailed experimental measurements (temperature and species compositions) at different oxygen levels in  
 196 the co-flow, ranging from 3% to 9%, allowing to control the system  $Da_\eta$  number, and at different fuel jet  
 197 Reynolds numbers (5000, 10000 and 20000). The MILD combustion burner (Figure 1) consists of a central  
 198 insulated fuel jet ( $\text{\O}4.6\text{mm}$ ) within an annular co-flow ( $\text{\O}82\text{mm}$ ) of hot exhaust products from a secondary  
 199 burner mounted upstream of the jet exit plane. The  $\text{O}_2$  level in the co-flow is controlled by the constant  
 200 flow-rate secondary porous burner. The ratio of the co-flow air/nitrogen was varied to give coflow  $\text{O}_2$  levels  
 201 of 3% (HM1), 6% (HM2) and 9% (HM3) (mass fractions), while the temperature and exit velocity were  
 202 kept constant at 1300 K and 3.2 m/s. The jet Reynolds number was varied for the 3%  $\text{O}_2$  level, from 5000  
 203 (HM1-5k) to 10000 (HM1) and 20000 (HM1-20k). The available data consist of mean and root mean square  
 204 (rms) of temperature and mass fractions of major ( $\text{CH}_4$ ,  $\text{H}_2$ ,  $\text{H}_2\text{O}$ ,  $\text{CO}_2$ ,  $\text{N}_2$  and  $\text{O}_2$ ) and minor species (NO,  
 CO and OH).

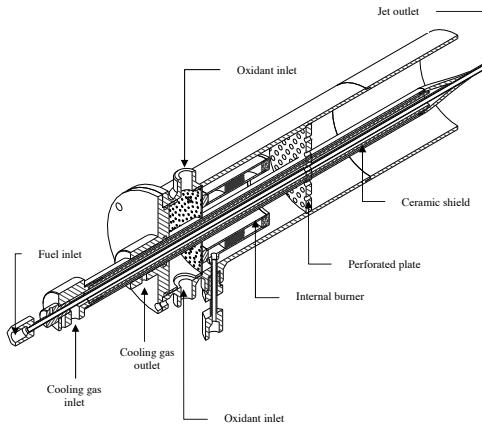


Figure 1: Schematic of the Jet in Hot Co-flow burner [19].

205

206 Numerical simulation were carried out using the Ansys FLUENT 14.5 CFD commercial code. A two-  
 207 dimensional steady-state simulation of the physical domain was considered due to the symmetry of the system.



208 The computational domain is 1000 mm in the axial direction and 120 mm in the radial direction from the  
 209 jet exit. The mesh is structured and non-uniform with about 20,000 cells, to provide high resolution in the  
 210 reaction zone and save computational effort elsewhere. Two additional meshes were considered to evaluate  
 211 the Grid Convergence Index (GCI), which gives a measure of deviation from the asymptotic numerical value  
 212 [42]. A GCI value of about 2% was obtained for temperature and major species, using the base grid and the  
 213 KEE-58 mechanism [43].

214 Velocity-inlet boundary conditions are specified at the inlets, whereas pressure-outlet conditions are ap-  
 215 plied at the boundaries assuming ambient air back-flow conditions, being the flame non-confined. Turbulence  
 216 is modelled using a modified  $k - \epsilon$  model, with the  $C_{\epsilon 1}$  parameter modified to a value of 1.60 for self-similar  
 217 round jets [44]. Particular attention was given to the specification of the turbulence level of the co-flow, as  
 218 previous studies [25, 27] indicated the very strong effect of turbulence intensity on the mixing level and the  
 219 quality of the predictions.

220 The KEE-58 mechanism is considered in the present study in combination with EDC model in its standard  
 221 formulation and using the modifications proposed in Section 2.2. The choice of the KEE-58 mechanism  
 222 was motivated by the relatively low computational cost associated with this mechanism compared to more  
 223 complete ones such as the GRI-2.11 and GRI-3.0 mechanisms [45, 46]. Figure 2 shows a comparison between  
 224 the measured and calculated temperature profiles, providing a benchmark of the three mechanisms. Results  
 225 show negligible differences between the KEE-58 and GRI-2.11/GRI-3.0 mechanisms, below 2-3% in all cases,  
 thus justifying the use of the less computational expensive KEE-58 scheme for the present investigation.

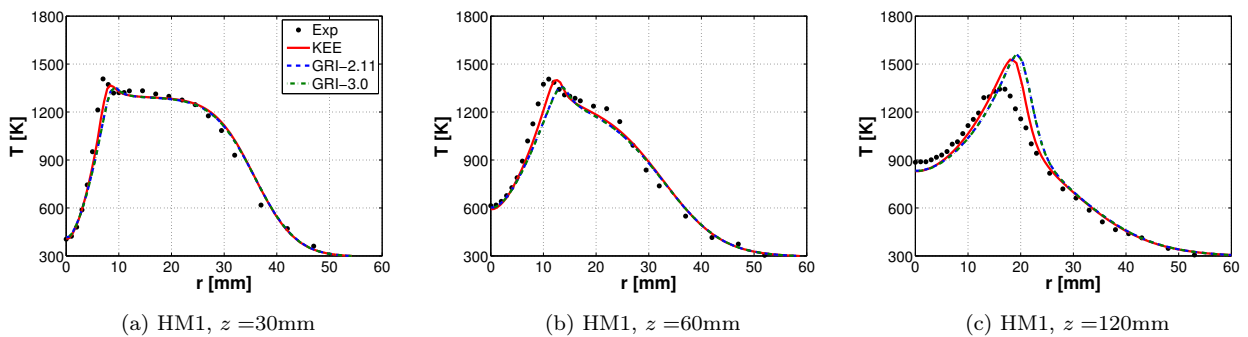


Figure 2: Comparison between measured and computed mean temperature for flame HM1 at three axial locations using the standard EDC with three chemical kinetic mechanisms, KEE-58 (red), GRI-2.11 (blue) and GRI-3.0 (green).

226

227 Differential diffusion is taken into account by calculating binary diffusion coefficients from the kinetic  
 228 theory, and the discrete ordinate (DO) method together with the Weighted-Sum-of-Gray-Gases (WSGG)  
 229 model is used for radiation. Second-order upwind schemes are employed for all equations and the SIMPLE  
 230 algorithm is used for pressure-velocity coupling.

231 *3.1. Determination of EDC coefficients*

232 The proposed approach for the determination of EDC parameters is tested in two different ways, using  
 233 global or local coefficients.

234 In the first approach, global coefficients are determined using Equations (18) and (21). The determination  
 235 of the modified model coefficients requires the knowledge of characteristic  $Re_T$  and  $Da_\eta$  numbers for the  
 236 system of interest.  $Re_T$  can be directly computed, based on the estimated values of the turbulent kinetic  
 237 energy,  $k$ , and dissipation rate,  $\epsilon$ , of the fuel jet,  $Re_T = k^2 / (\nu\epsilon)$ . On the other hand, the estimation of  $Da_\eta$   
 238 requires, in principle, the *a priori* knowledge of the solution. Isaac et al. [41] recently developed an approach  
 239 for the calculation of chemical time-scales of turbulent combustion data with detailed chemistry, based on  
 240 the down-sizing of the chemical source term Jacobian using Principal Component Analysis. The analysis was  
 241 demonstrated on the HM1, HM2 and HM3 flames, for which values of  $Da_\eta$  of 0.775, 1.15 and 1.4 [41] were  
 242 estimated, respectively. Therefore, the availability of characteristic  $Re_T$  and  $Da_\eta$  numbers allow us to find  
 243 the values of  $C_\tau$  and  $C_\gamma$ , as indicated in Table 1. For the constant Reynolds number (10,000), oxygen varying  
 244 flames (HM1, HM2 and HM3), the variation of the coefficients  $C_\tau$  and  $C_\gamma$  is only due to the variation of  $Da_\eta$   
 245 . On the other hand, for the Reynolds varying HM1 flames at  $y_{O_2} = 0.03$  (HM1-5k, HM1 and HM1-20k),  
 246 both  $Re_T$  and  $Da_\eta$  change (due to the change in turbulent dissipation rate, being  $\tau_\eta = (\nu/\epsilon)^{\frac{1}{2}}$ ), as indicated  
 247 in Table 1. It can be observed that the approach requires the determination of a reference case, for which  
 248 standard values of EDC coefficients are applied. For  $C_\tau$ , the reference was set through extrapolation of  
 249 the  $Da_\eta$  values from diluted ( $y_{O_2} = 0.03$ ) to standard air ( $y_{O_2} = 0.232$ ) conditions, using the estimated  
 250  $Da_\eta$  values for HM1-HM3 flames. This gave a reference  $Da_\eta$  of 2.8, at which  $C_\tau = 0.4083$ . In the present  
 251 investigation, the focus is initially put on the variation of the coefficient  $C_\tau$ . This allows to better assess the  
 252 soundness of the proposed approach, since the observed trends can be more easily explained and linked to a  
 253 single parameter variation. The effect of the simultaneous variation of  $C_\tau$  and  $C_\gamma$  is also investigated.

Table 1:  $C_\tau$  and  $C_\gamma$  as a function of global  $Re_T$  and  $Da_\eta$  values.

Flame	$Re_T$	$Da_\eta$	$C_\tau$	$C_\gamma$	Flame	$Re_T$	$Da_\eta$	$C_\tau$	$C_\gamma$
HM1	400	0.78	1.47	1.90	HM1-5k	225	1.20	1.25	1.78
HM2	400	1.15	1.00	2.14	HM1	400	0.78	1.47	1.90
HM3	400	1.4	0.82	2.14	HM1-20k	760	0.50	1.77	2.00

254 When using local coefficients, the values of  $Re_T$  and  $Da_\eta$  must be computed locally for each cell. The  
 255 procedure is straightforward for the turbulent Reynolds number, which can be easily computed using the  
 256 local value of turbulent kinetic energy and turbulent dissipation rate.. As far as the Damköhler number is  
 257 concerned, its evaluation requires the estimation of the leading chemical time scale,  $\tau_c$ , through the analysis  
 258 and decomposition of the Jacobian of the system chemical source terms [41]. Such a procedure can be very  
 259 expensive for the on-the-fly determination of the EDC parameters. Therefore, the estimation of the controlling

260 chemical time scale was based on a one-step chemistry [47], obtaining temperature and the necessary species  
 261 concentrations from the detailed chemical mechanism. This approach was found to provide reasonable good  
 262 predictions for the system under investigation in [41], being the one-step chemistry applied on accurate  
 263 thermal and concentration fields. Once  $\tau_c$ , is available,  $Da_\eta$  is simply computed from the definition  $Da_\eta =$   
 264  $\tau_\eta/\tau_c$ , where  $\tau_\eta = (\nu/\epsilon)^{\frac{1}{2}}$ . The use of local coefficients required the modification of original EDC model.  
 265 Indeed, Ansys FLUENT does not allow the change of the coefficients locally. Therefore, two User-Defined  
 266 Functions (UDF) were used: one for the calculation of the EDC coefficients based on local values of the  
 267 Reynolds and Damköhler numbers; and the second to compute the net formation rate for each species  
 268 based on the EDC formulation. A contour plot showing the  $Re_T$  and  $Da_\eta$  numbers for the system under  
 investigation is shown in Figure 3. The typical distribution of  $C_\tau$  and  $C_\gamma$  for the flame under investigation

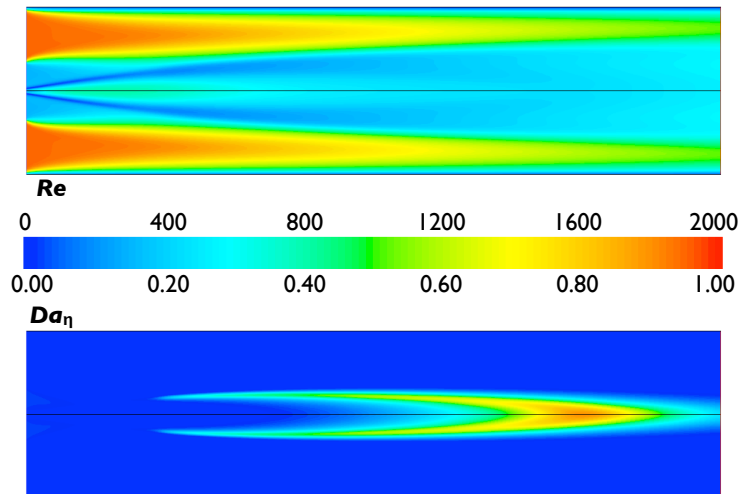


Figure 3: Characteristic  $Re_T$  and  $Da_\eta$  distribution for the JHC system. The horizontal axis denotes the axial direction.  $O_2$  level in the co-flow: 3%. Fuel-jet Reynolds number:10,000.

269  
 270 is shown in Figure 4. The same formulation of the local coefficients was employed for all calculations, to  
 271 avoid any fitting/optimization of the results for specific operating conditions. Figure 4a-b shows that the  
 272 EDC local coefficients are limited to their standard values, i.e.  $C_\gamma = 2.1377$  and  $C_\tau = 0.4083$ . Figure 4c  
 273 shows the  $Re^*$  from the modified local coefficients, indicating value in the range 3-5 in the ignition region,  
 274 in agreement with the modified EDC model recently proposed by Evans et al. [29]. The distribution of  $C_\gamma$   
 275 (Figure 4b) is due to the fact that for very low values of  $Da_\eta$ , the  $C_\gamma$  was set to its standard value, to ensure  
 276 proper ignition. The threshold value for  $Da_\eta$  was set to 0.01 in the present work; however, it was verified  
 277 that the results were insensitive to a variation of 50% around this value, to ensure the robustness of such  
 278 a choice. The need of a threshold value is mostly related to the approach used for the calculation of  $Da_\eta$ ,  
 279 based on a one-step chemistry. This is very practical for the on-the-fly calculation of the model coefficients,  
 280 but it results in unrealistic estimations of  $Da_\eta$  values outside of the flame region.

281 **4. Results**

282 This section describes the results obtained for the test cases at varying co-flow concentrations and fuel-  
 283 jet Reynolds numbers, with the objective of assessing the effect of the proposed modification of the EDC  
 284 coefficients on the results. First, the results of the approach based on global coefficients will be presented and  
 285 discussed. Then, the results of the local coefficients approach will be shown. To better assess the quantitative  
 286 agreement between model predictions and measurements, the results shown in the present section do not only  
 287 indicate the mean observed value of the scalar under consideration (temperature and species mass fractions)  
 288 but also the 95% confidence for the true mean value,  $\mu$ , associated to the measurements,  $\bar{y}_e$ , calculated as  
 289 [48]:

$$\bar{y}_e - t_{\frac{\alpha}{2}, \nu} \frac{s}{\sqrt{n}} < \mu < \bar{y}_e + t_{\frac{\alpha}{2}, \nu} \frac{s}{\sqrt{n}} \quad (22)$$

290 where  $t_{\frac{\alpha}{2}, \nu}$  is the  $(1 - \frac{\alpha}{2})$  quantile of Student's t-distribution defined by the  $n$  experimental observations.  
 291 with  $\nu = n - 1$  degrees of freedom, and  $s$  is the sample standard deviation,

$$s = \left[ \frac{1}{n-1} \sum_{i=1}^n (y_e^i - \bar{y}_e)^2 \right]^{\frac{1}{2}}. \quad (23)$$

292 The calculation of the confidence intervals was made possible by the availability of a large number of obser-  
 293 vations  $y_e^i$  ( $\sim 500$ ) for each measurement point.

294 *4.1. Modified EDC - global coefficients*

295 Figure 5 shows the radial temperature profiles at different axial locations, for flames HM1 (a-c), HM2  
 296 (d-f) and HM3 (g-i). The modified EDC results shown are obtained through modification of the coefficient  
 297  $C_\tau$ . It can be observed that the adjustment of  $C_\tau$  determines a generalized improvement of predictions: the  
 298 temperature over-prediction is strongly reduced and the radial temperature distribution is better captured.  
 299 At high axial distances, i.e.  $z = 120$  mm, the behaviour of the model remains unsatisfactory, although  
 300 an improvement is noticeable with respect to the standard EDC model. To further confirm the qualitative  
 301 analysis based on the observation of Figure 5, the relative error in the prediction of the maximum temperature  
 302 at different axial locations is shown in Table 2. The peak temperature for the HM1 flame at  $z = 120$  mm  
 303 decreased from 1716 K to about 1526 K, the experimental value being 1343 K, implying a reduction of the  
 304 relative error from 28% to 14%. Similar improvements are observed for HM2 and HM3 flames, for which the  
 305 error decreases from to 26% to 19% and from 20% to 16%, respectively. The trend is confirmed at all axial  
 306 distances and for all flames, with the exception of location  $z = 30$  mm for HM1 flame, where the performances  
 307 of standard and modified models are comparable. In particular, the modified model performs remarkably  
 308 well at  $z = 60$  mm when compared to standard EDC settings. Interestingly, the coefficient modification  
 309 appears more beneficial as the departure from conventional combustion conditions is more important. This  
 310 is of straightforward interpretation, being the developed model based on theoretical considerations valid in

Table 2: Relative error on maximum temperatures at different axial locations of standard and modified EDC models for flames at varying oxygen concentration (HM1-HM3).

		HM1, Relative error on $T_{max}$ [%]		
$z$ [mm]	Std	Global coefficients		Local coefficients
		$C_\tau$	$C_\tau$ & $C_\gamma$	
30	0.91	2.90	3.80	3.48
60	5.00	0.50	3.60	1.75
120	27.80	13.60	5.60	9.35
		HM2, Relative error on $T_{max}$ [%]		
$z$ [mm]	Std	Global coefficients		Local coefficients
30	8.20	4.00		1.84
60	10.80	6.10		4.37
120	25.90	18.50		2.27
		HM3, Relative error on $T_{max}$ [%]		
$z$ [mm]	Std	Global coefficients		Local coefficients
30	6.00	2.40		8.56
60	8.50	5.10		10.49
120	20.30	15.50		3.08

the framework of distributed reaction regime limit. The quantitative results shown in Table 2 support the proposed modification of the EDC coefficients and provide a theoretical basis to previous results obtained by other authors [26–28].

The availability of experimental data at different fuel-jet Reynolds numbers allows evaluating the performances of the proposed model when modifying the Reynolds number at the conditions of highest dilution ( $y_{O_2} = 0.03$ ). The fuel-jet Reynolds number is varied from 5,000 (HM1-5k) to 20,000 (HM1-20k), resulting in different characteristic turbulent Reynolds numbers with respect to the base case (HM1) (Table 1). Figure 7 shows the radial temperature profiles at different axial locations, for flames HM1-5k (a-c) and HM1-20k (d-f). Results confirm the trend observed for the varying oxygen cases, indicating that the modified model yields improved predictions of temperature distribution at both Reynolds numbers. This is also proved by the quantitative results in Table 3, which shows in particular remarkable performances for the HM1-5k flame.

The HM1-20k flame deserves a separate discussion. The experimental data show a strong temperature reduction for increasing distance from the burner nozzle, at  $z = 120$  mm, due to the partial extinction of the flame caused by the increased jet velocity. This phenomenon can be clearly observed in Figure 6, which shows the instantaneous temperature measurements as a function of the mixture fraction for the HM1 flames at Reynolds numbers 10,000 (a) and 20,000 (b), respectively. The amount of partial extinction and re-ignition is significantly higher at higher Reynolds number, as indicated by the large data scatter in Figure

328 6b compared to Figure 6a. This phenomenon is not captured by the standard EDC formulation, which  
 329 results in a significant over-prediction of the temperature levels at  $z = 60$  mm and  $z = 120$  mm (Table 3).  
 330 The modification of the EDC model coefficients improves the model predictions, as indicated by the error  
 331 metrics in Table 3 and the radial temperature distributions in Figure 7d-f. However, with the modified EDC  
 332 formulation, the flame extinguishes for axial distances higher than  $z = 120$  mm and the model is unable to  
 333 reproduce the re-ignition observed experimentally. This indicates that the combination of RANS modelling  
 334 with the EDC approach is not adequate to model the HM1-20k flame, as the model provides either a stable  
 335 flame with temperature levels significantly higher than those observed experimentally or an extinguishing but  
 336 not re-ignition flame. It was attempted to model the HM1-20k flame using the transported PDF approach,  
 337 but this also resulted in global extinction, as indicated in the literature [49]. This suggests that more complex  
 approaches, i.e. unsteady simulations and/or LES, should be employed to model such a flame.

Table 3: Relative error on maximum temperatures at different axial locations of standard and modified EDC models for flames at varying fuel-jet  $Re$  numbers (HM1-5k and HM1-20k).

$z$ [mm]	HM1-5k, Relative error on $T_{max}$ [%]	
	$C_\tau=0.4083$	$C_\tau=1.96$
30	5.90	1.86
60	11.80	3.31
120	20.90	10.35
$z$ [mm]	HM1-20k, Relative error on $T_{max}$ [%]	
	$C_\tau=0.4083$	$C_\tau=1.07$
30	0.74	2.95
60	16.00	3.21
120	82.20	10.22

338

339 The effect of the simultaneous change of the two model coefficients,  $C_\tau$  and  $C_\gamma$ , on the results was also  
 340 investigated. Figure 8 shows the radial temperature profiles at  $z = 30$  mm (a),  $z = 60$  mm (b) and  $z = 120$   
 341 mm (c) for the HM1 flames, providing a benchmark between all the tested models. It appears evident that  
 342 the modification of  $C_\gamma$  has a minor influence at axial distances  $z < 60$  mm, whereas a clear effect can  
 343 be observed at  $z = 120$  mm, where the over-prediction of the temperature distribution is further reduced,  
 344 leading to better agreement between measurements and simulations with respect to the case based on the  
 345 modification of the coefficient  $C_\tau$ . This is confirmed quantitatively by the results in the first columns of Table  
 346 2, which shows a relative error metric for maximum temperature decreasing from 13.6% to about 5%, when  
 347 modifying simultaneously the two model coefficients.

348 To further assess the potential improvement associated with the use of modified coefficients, the standard  
 349 and modified formulation are compared against the experimental observation of major ( $\text{CO}_2$  and  $\text{H}_2\text{O}$ ) and  
 350 minor species ( $\text{CO}$ ) and radicals ( $\text{OH}$ ). Figure 9 shows the radial profile of  $\text{CO}$  mass fraction at different

axial locations for the HM1 (a-c), HM2 (d-f) and HM3 (g-i) flames, using the standard EDC formulation and the modified one with global coefficients. The observed improvement in temperature distributions is expected to yield better agreement for observed and measured CO mass fractions. At  $z = 30$  mm for the HM1 case (Figure 9a), the modified model formulation provides better prediction only far from the axis, underestimating the CO level at the centerline with respect to the standard model. On the other hand, the improvements are significant and more clearly visible for the HM2 and HM3 (Figure 9d and Figure 9g respectively), particularly at the centerline. The same conclusions can be drawn for the CO profiles at  $z = 60$  mm and at  $z = 120$  mm. A similar analysis for the OH radical (Figure 10) shows that the use of the modified coefficients provides improved results for increasing distances from the burner exit, whereas close to the burner the OH peak is better captured by the standard EDC model. Moreover, it is possible to observe that both default and modified models perform poorly at  $z = 120$  mm. We mainly attribute this behaviour to the intermittent localized flame extinction, documented in the literature for the flames under investigation [24, 27], especially at diluted conditions. While the EDC model is capable of capturing the flame lift-off, it fails in capturing such non-equilibrium phenomena.

The radial profiles of major species, i.e.  $\text{CO}_2$  and  $\text{H}_2\text{O}$ , indicate that the use of the modified model always results in better predictions than the standard model. Figures 11 and 12 indicate that the modified model better captures the observed values close to the axis, as well as at the peak value and values far from the axis.

It is important to note that the analysis of the results cannot be only limited to the role of the modified EDC coefficients, but it should include further investigations on the influence of the turbulence model and the kinetic mechanism. The objective of the present paper is to assess, for a given configuration, if the proposed model for the modification of the EDC coefficients results in explainable and consistent trends, which appears evident from the results.

#### 4.2. Modified EDC - local coefficients

Finally, we analyzed the effect of local coefficients on the results. The objective is to verify that the on-the-fly calculation of the EDC coefficients provides results in agreement with those obtained using the modified global coefficients, without the need for an *a priori* knowledge of the results, which would make the practical use of the theory developed in the present paper quite challenging. Figure 13 shows the radial temperature profiles at different axial locations, for flames HM1 (a-c), HM2 (d-f), HM3 (g-i) and HM1-5k (j-l), using the modified EDC formulation with globally and locally (in each grid cell) computed coefficients, respectively. One general observation that can be made from these plots is that the results obtained with a local evaluation of the coefficients compares very well with the results obtained using modified global coefficients, providing thus improved results with respect to the standard EDC formulation. In some cases, far from the burner nozzles, the results obtained using the local coefficients show even better agreement with the experimental data. The same conclusions hold also for species distributions. In particular, the OH radial profiles at  $z = 120$  mm for flames HM1-HM3 (Figure 14) indicate that, while the results using global and

387 local coefficients are very similar for the HM1 flame, there is a substantial improvement in OH prediction  
388 when adopting local coefficients. This indicates that for the very diluted conditions (HM1), the use of the  
389 local coefficients does not solve the problem related to the inability of EDC of capturing the localized flame  
390 extinction. However, for the less diluted, more stable conditions (HM2-HM3), a local evaluation of the EDC  
391 coefficients gives improved results in the contracted region of the flame, where the use of global constants  
392 (tuned using the fuel jet characteristics) over-estimates the reaction rates.

393 We can therefore conclude that the use of local coefficients for EDC is a viable option for the practical  
394 implementation of the proposed functional dependencies between the EDC coefficients and the dimensionless  
395  $Re_T$  and  $Da_\eta$  numbers. The use of local coefficients has clear advantages over the use of global ones,  
396 as it does not require prior knowledge of the characteristic dimensionless numbers ( $Re_T$  and  $Da_\eta$ ) in the  
397 system. Moreover, it should be stressed out that the use of local coefficients does not have an impact on the  
398 simulation time, as the latter is dominated by the chemistry integration time. This method can thus be seen  
399 as an effective way for on-the-fly calculations.

## 400 5. Conclusions

401 The Eddy Dissipation Concept (EDC) for turbulent combustion is widely used to model turbulent reacting  
402 flows where chemical kinetics may play an important role, as it is the case for MILD combustion. However,  
403 recent investigations have pointed out the limitations of the approach for such a regime. The present paper  
404 proposes a modification of the EDC model coefficients to allow its application in the context of MILD  
405 conditions. The main findings of the present work can be summarized as follows:

- 406 • The energy cascade model, on which EDC is founded, was revisited in the limit of the distributed  
407 reaction regime, to derive explicit dependencies between the EDC model coefficients and the Reynolds  
408 and Damköhler dimensionless numbers.
- 409 • The proposed approach was validated on several data sets collected on the JHC burner with different  
410 co-flow composition (3%, 6% et 9% O<sub>2</sub> mass fraction) and fuel-jet Reynolds numbers (5,000, 10,000  
411 and 20,000). For the 20,000 Reynolds number case, the modification of the EDC model coefficients  
412 improves the model predictions close to the burner. However, the model is unable to reproduce the  
413 re-ignition observed experimentally at higher axial distance. This does not appear related to the EDC  
414 model itself, rather to the RANS modelling limitations.
- 415 • The proposed approach was first validated using modified global coefficients, determined on the ba-  
416 sis on available simulation results. Results based on the variation of the time scale coefficient,  $C_\tau$ ,  
417 show promising improvement with respect to the standard EDC formulation close to the burner, espe-  
418 cially at diluted conditions and medium to low Reynolds numbers, for both temperature and species  
419 measurements. Moreover, the simultaneous modification of the time scale,  $C_\tau$ , and the mass fraction



420 coefficients,  $C_\gamma$ , leads to improvements in the model predictions at large axial distances from the burner  
421 exit.

422 • The proposed approach was then validated using a local evaluation of the EDC model coefficients,  
423 using the local values of  $Re_T$  and  $Da_\eta$  numbers. The  $Re_T$  value was estimated from the local values  
424 of turbulent kinetic energy and dissipation rate, while  $Da_\eta$  was obtained from a one-step reaction,  
425 using the temperature and species concentrations coming from the detailed mechanism employed for  
426 the gas-phase reactions. Results from the local approach are comparable or superior to those provided  
427 via the modification of the global coefficients, thus indicating the viability of the approach.

428 Future work will investigate the influence of the turbulence model and kinetic mechanism on the predictions  
429 and their impact on the modified coefficients, using validation data from experiments and Direct Numerical  
430 Simulations. Moreover, more accurate approaches for the determination of the chemical time-scale and  $Da_\eta$   
431 number will be investigated.

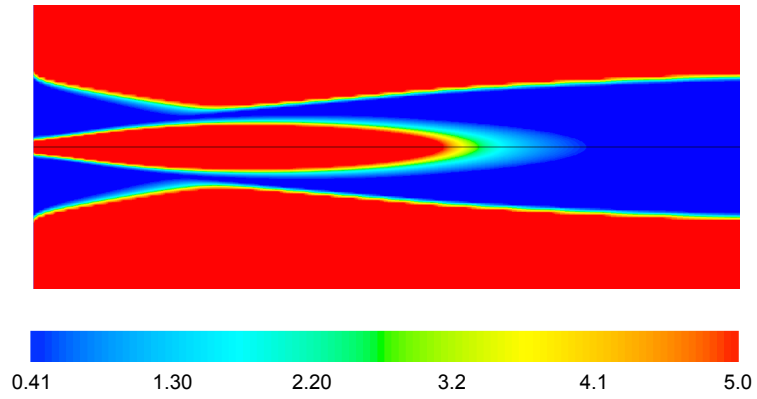
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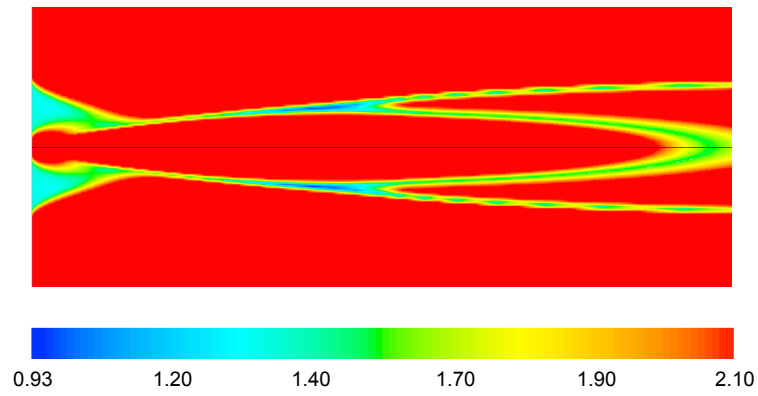
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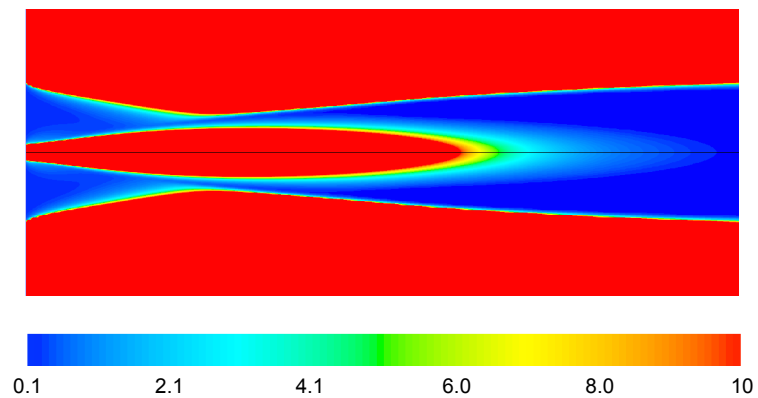
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(a)  $C_\tau$



(b)  $C_\gamma$



(c)  $Re^*$

Figure 4: Characteristic  $C_\tau$  (a),  $C_\gamma$  (b) and  $Re^*$  (c) distribution for the JHC system. The horizontal axis denotes the axial direction.  $O_2$  level in the co-flow:3%. Fuel-jet Reynolds number:10,000.

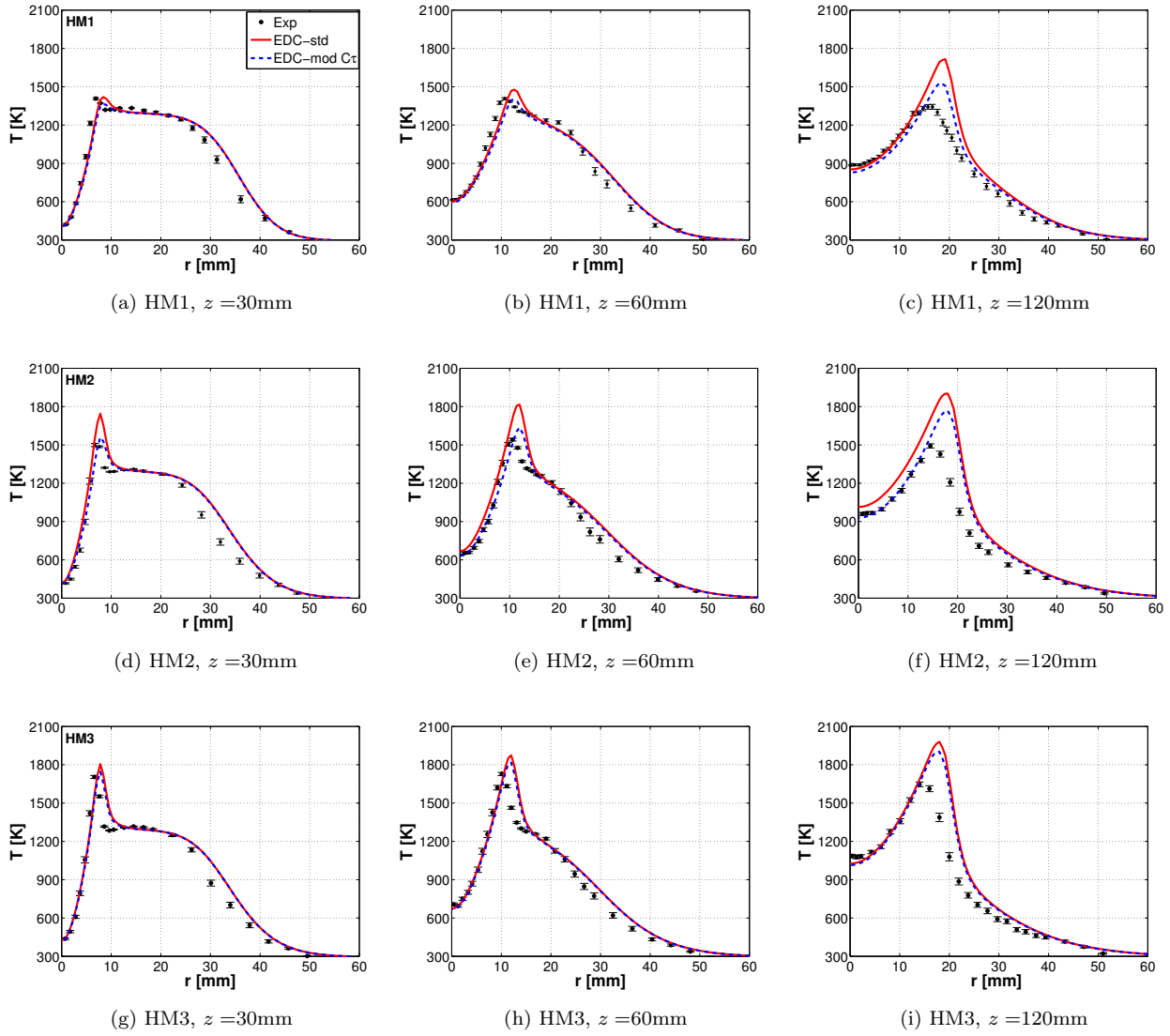


Figure 5: Comparison of measured and computed radial temperature profiles at different axial locations, for flames HM1 (a-c), HM2 (d-f) and HM3 (g-i).

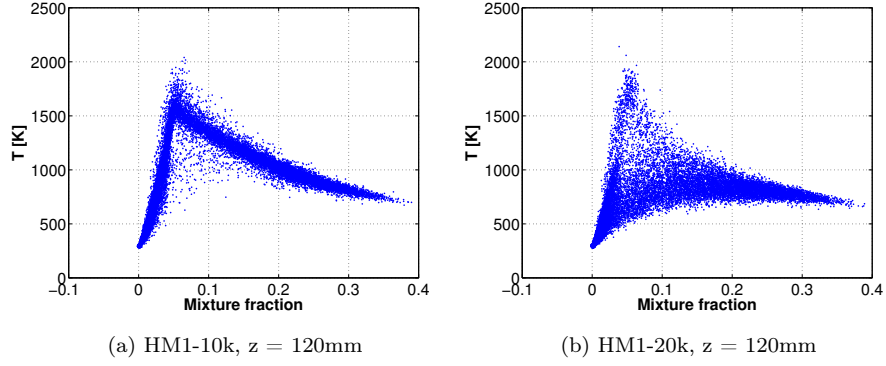


Figure 6: Scatter plots of temperature as a function of the mixture fraction for the  $Re=10,000$  (HM1-10k) (a) and  $Re=20,000$  (HM1-20k) (b) flames.

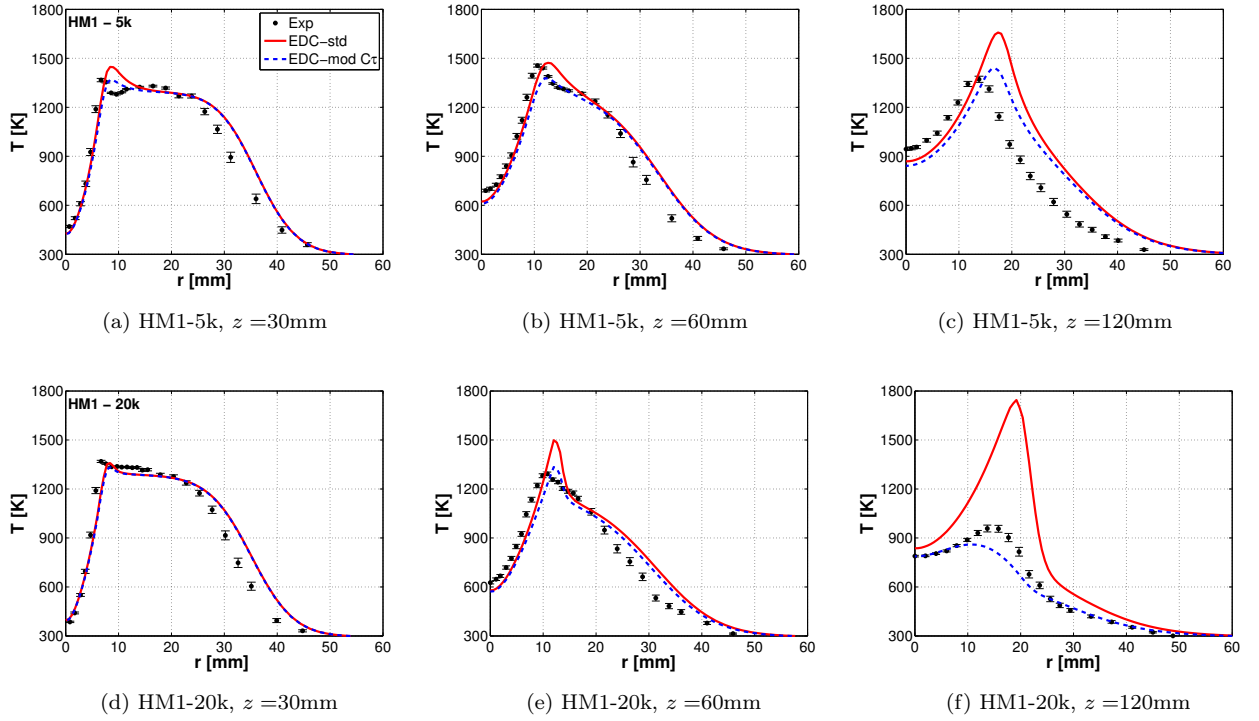


Figure 7: Radial temperature profiles at different axial locations along the axis, for flames HM1-5K (a-c) and HM1-20K (d-f).

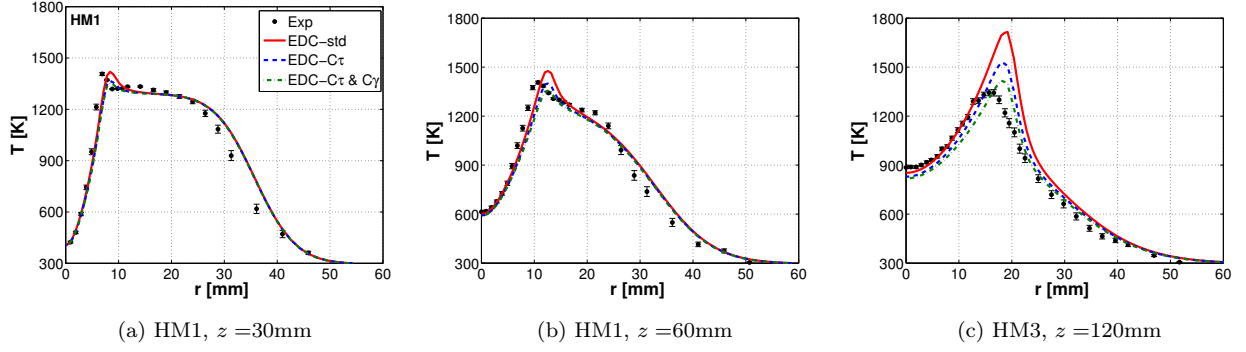


Figure 8: Radial temperature profiles at different axial locations along the axis for flame HM1, resulting from the simultaneous modification of the EDC coefficients.

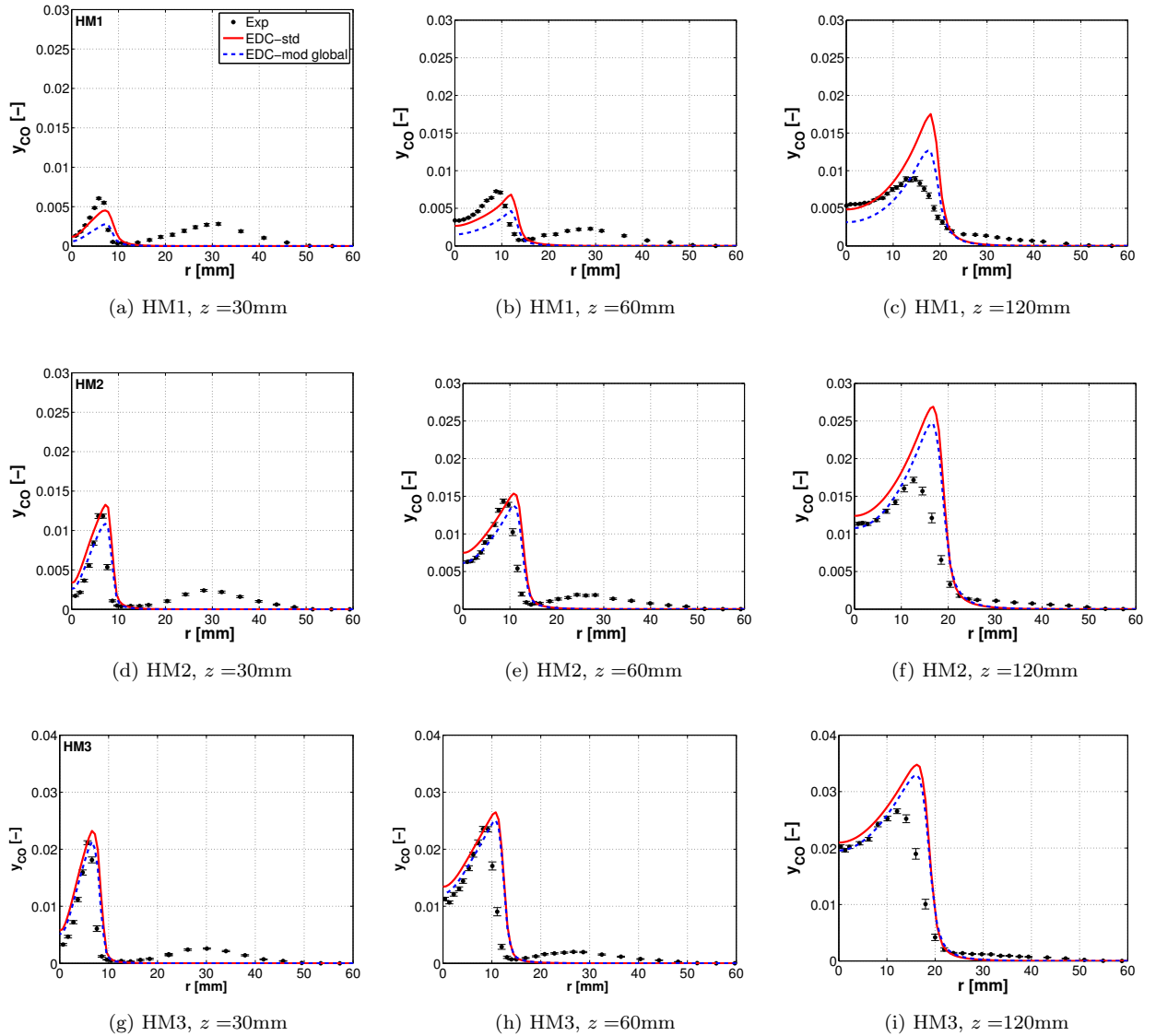


Figure 9: Radial profile of CO mass fraction at different axial locations, for flames HM1 (a-c), HM2 (d-f) and HM3 (g-i).

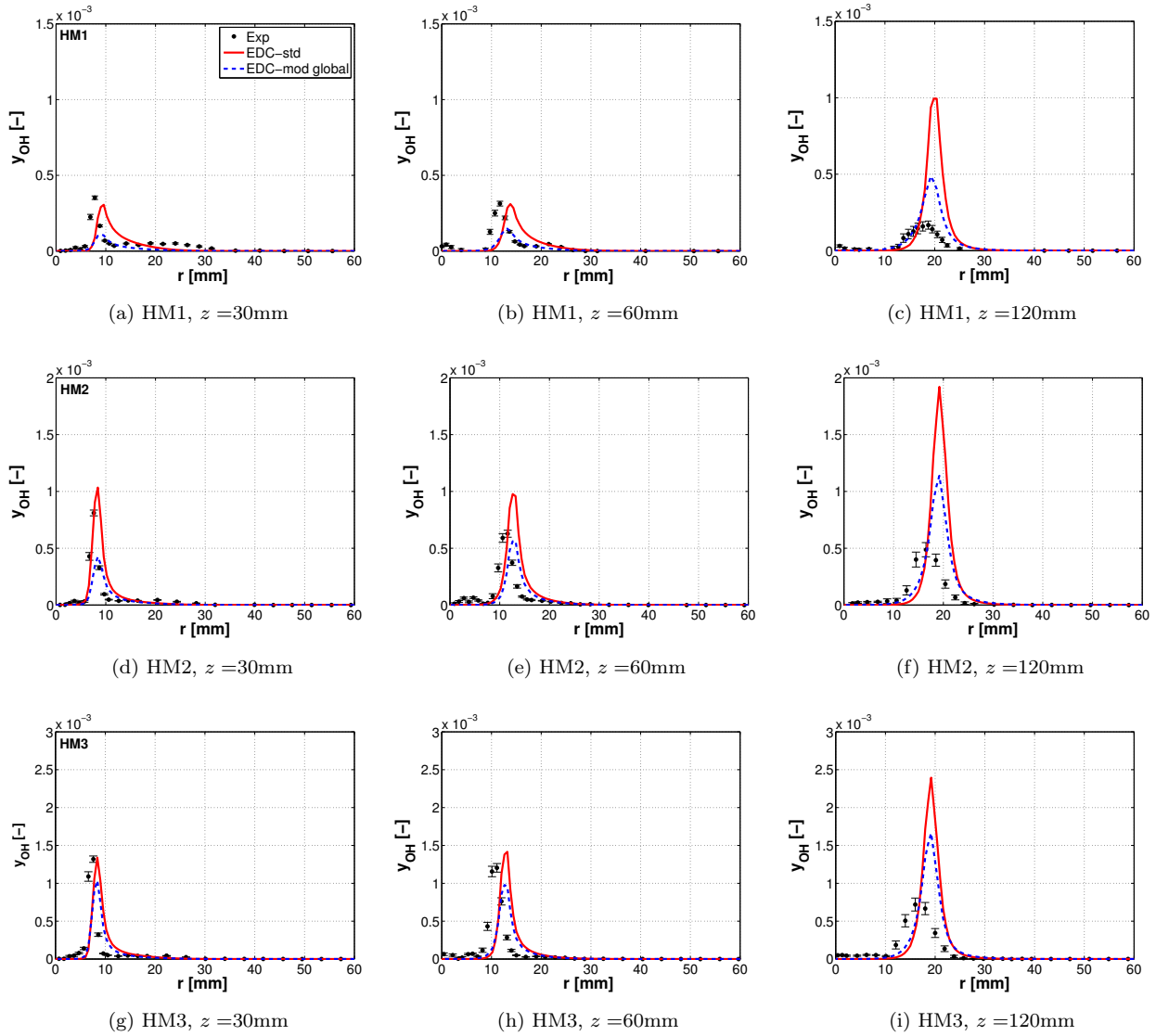


Figure 10: Radial profile of OH mass fraction at different axial locations, for flames HM1 (a-c), HM2 (d-f) and HM3 (g-i).



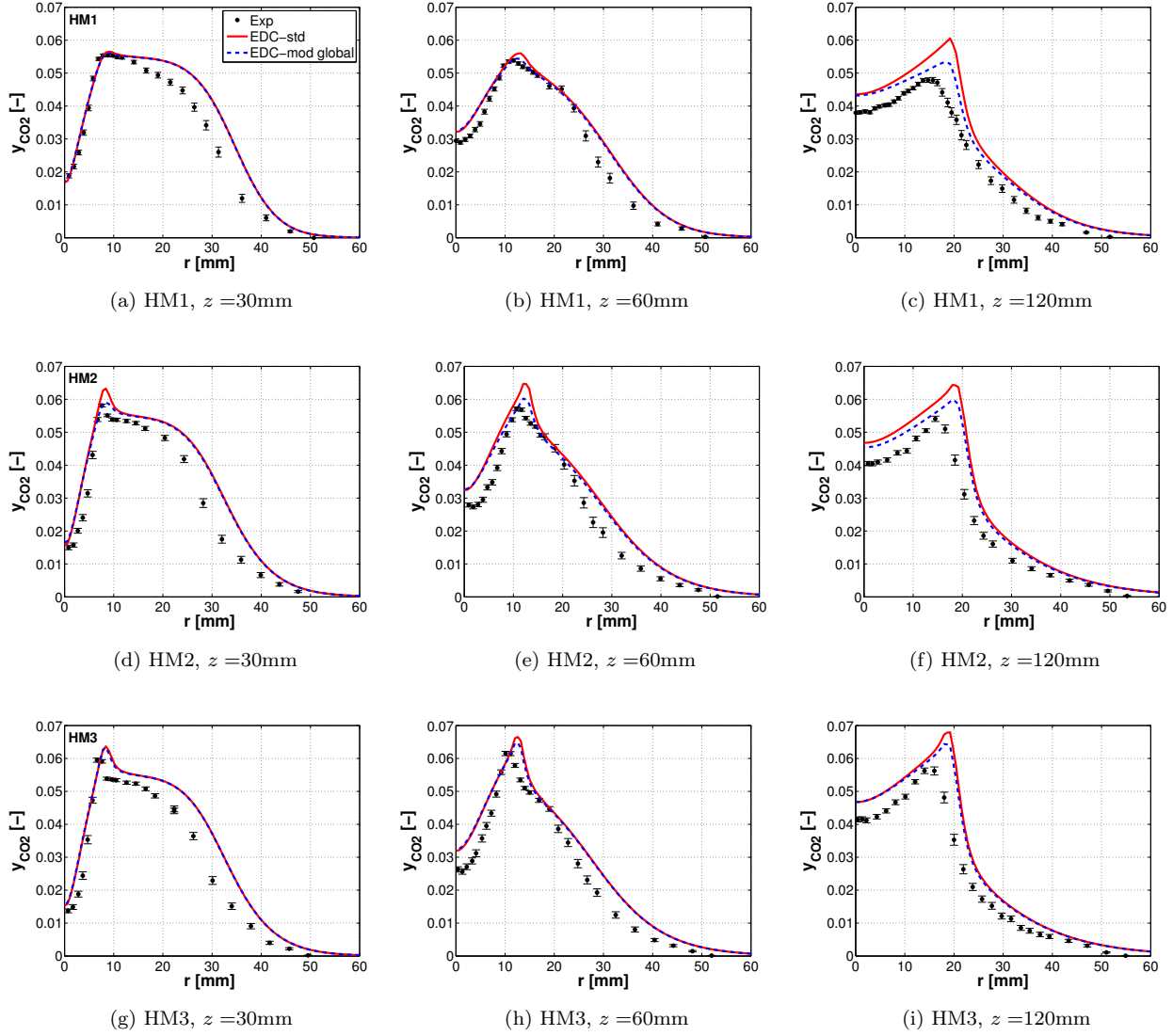
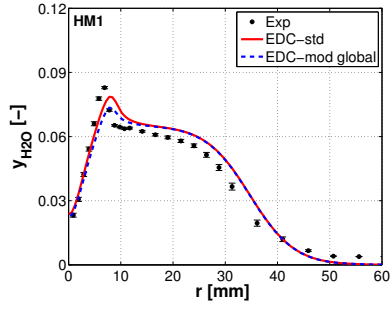
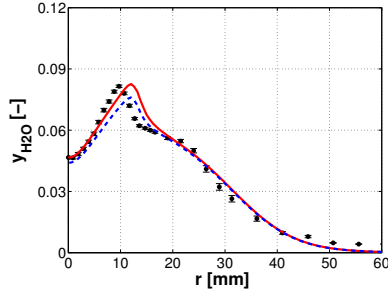


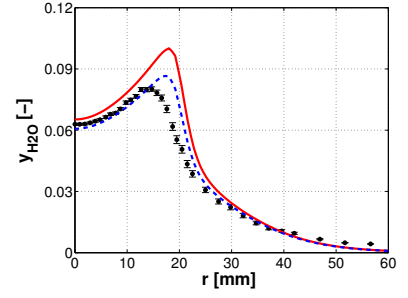
Figure 11: Radial profile of CO<sub>2</sub> mass fraction at different axial locations, for flames HM1 (a-c), HM2 (d-f) and HM3 (g-i).



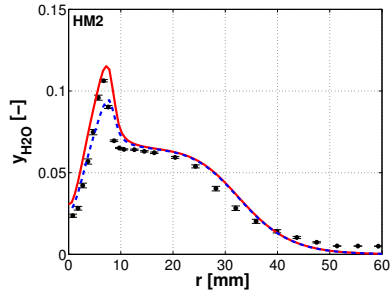
(a) HM1,  $z = 30\text{mm}$



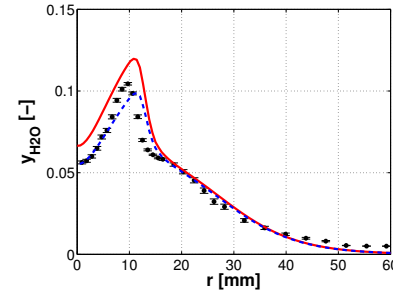
(b) HM1,  $z = 60\text{mm}$



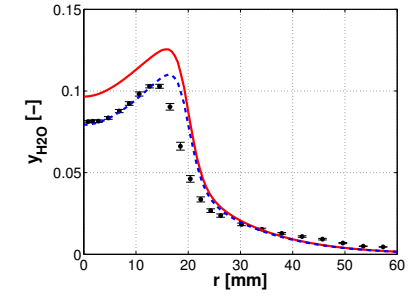
(c) HM1,  $z = 120\text{mm}$



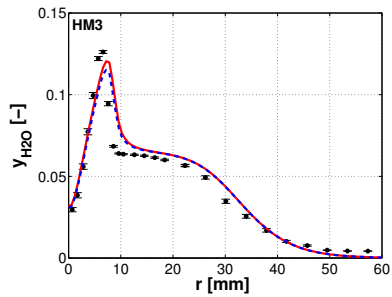
(d) HM2,  $z = 30\text{mm}$



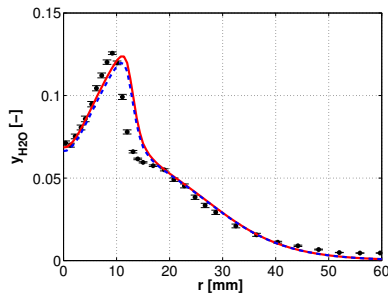
(e) HM2,  $z = 60\text{mm}$



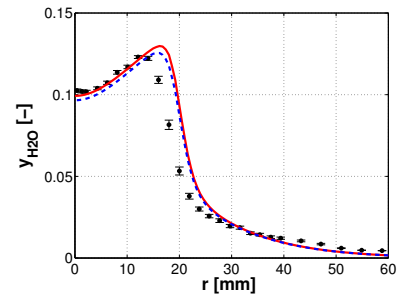
(f) HM2,  $z = 120\text{mm}$



(g) HM3,  $z = 30\text{mm}$



(h) HM3,  $z = 60\text{mm}$



(i) HM3,  $z = 120\text{mm}$

Figure 12: Radial profile of H<sub>2</sub>O mass fraction at different axial locations, for flames HM1 (a-c), HM2 (d-f) and HM3 (g-i).

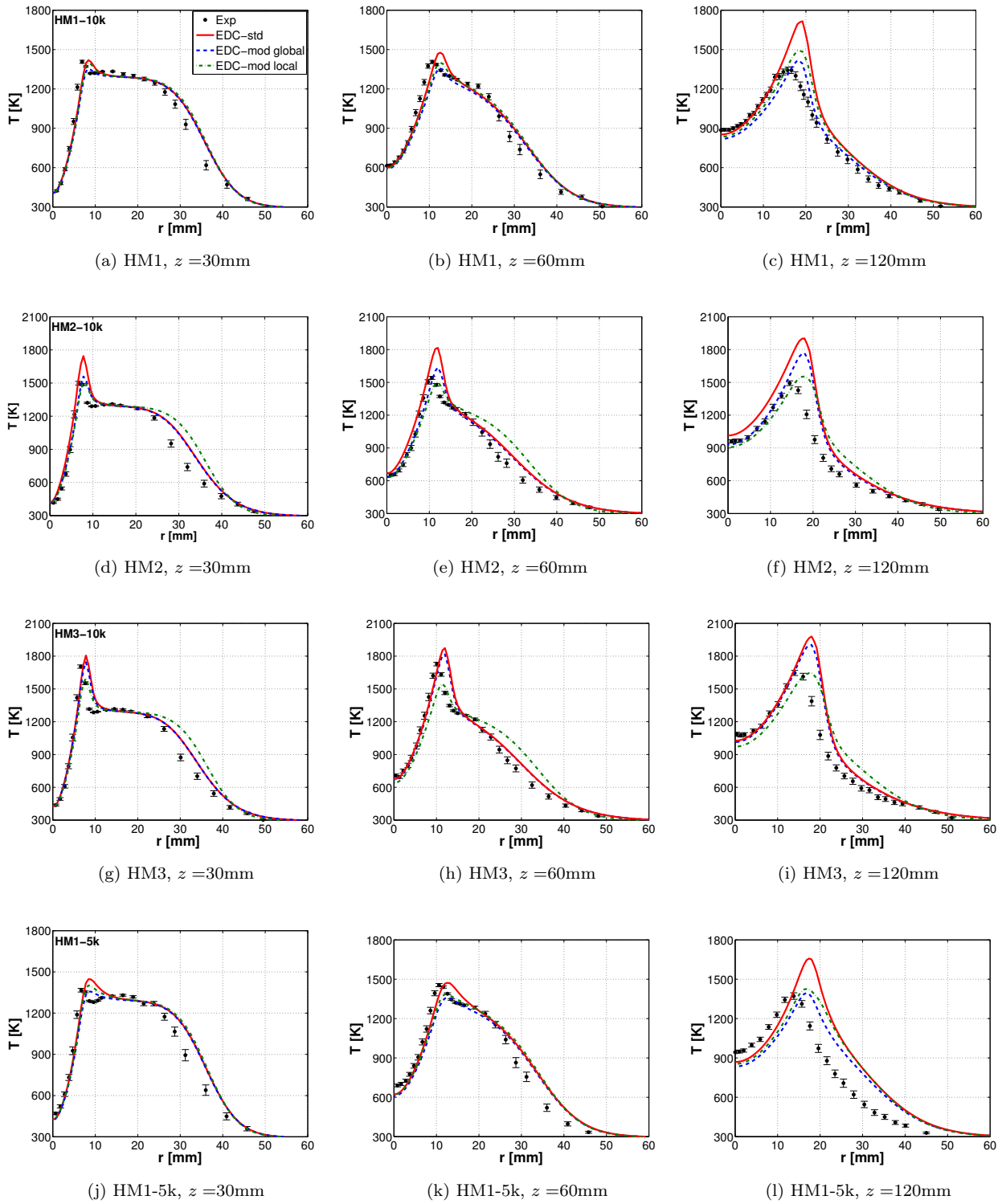


Figure 13: Radial temperature profiles at different axial locations, for flames HM1 (a-c), HM2 (d-f), HM3 (g-i) and HM1-5k (j-l).

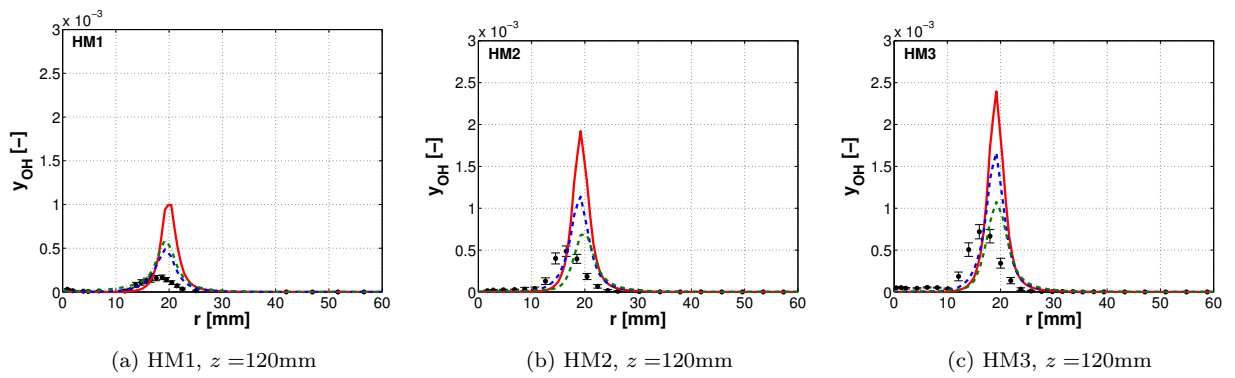


Figure 14: Radial profiles of OH mass fraction at  $z = 120$  mm, for flames HM1 (a), HM2 (b) and HM3 (c).