Reprinted from

THE JOURNAL OF EGYPTIAN ARCHAEOLOGY

VOLUME 85 1999

BRIEF COMMUNICATIONS

Note on the pyramidion found at Dahshur

The significant discrepancy between the slope of the pyramidion found at Dahshur and the slope of the Red Pyramid, beside which it was found, suggests that this pyramidion might have been planned for another pyramid. Study of surviving pyramidia and the evidence provided by the other pyramids in the area seem to point to the second stage of the construction of the Bent Pyramid as its original destination.

IN 1982, the expedition of the Deutsches Archäologisches Institut Kairo at Dahshur discovered fragments of an uninscribed limestone pyramidion among the debris surrounding the Red Pyramid.¹ Although still in pieces, the pyramidion immediately appeared to be steeper than the pyramid. The discrepancy was ascribed to a deliberate variation of the slope of the pyramid during construction, in order to make the uppermost part of the monument more visible from the ground.² Since then, the fragments have been assembled and placed in front of the pyramid. The result of the reconstruction, however, raises questions about the relationship between this pyramidion and the Red Pyramid.

Because of the large number of fragments and the generous use of plaster in the reconstruction, the surfaces of the pyramidion are slightly irregular. Nevertheless, at about 96 cm from the top, measured along the edge, it is possible to measure the breadth of the faces directly on the original pieces. They are about 96 cm wide, which means that the four faces were each equilateral triangles. The pyramidia of Amenemhat III (at Dahshur) and Khendjer, both in the Cairo Museum, show the same proportions, that is, the length of the edge is equal to the length of the base.³

The Egyptians measured the slope, which was called *seked*, as the horizontal displacement of the sloping face for a vertical drop of one cubit.⁴ That is, they measured the number of cubits, palms and fingers from which the sloping side had 'moved' from a vertical line at the height of one cubit. Basically the Egyptians constructed a right-angled triangle: of the two *catheti* (i.e. the two sides at a right angle to one another), one was equal to one cubit, the other corresponded to the *seked* (fig. 1).⁵ In a pyramid, if edges and base have the same length, the four faces are four equilateral triangles resting on oblique planes inclined toward the vertical axis. The slope of such a pyramid can be measured as 54°30', or a *seked* of 5 palms (fig. 1b). Although it does not seem to have been a very regular piece altogether,⁶ the pyramidion found at Dahshur appears to have shared these geometrical characteristics.

Of the casing of the pyramids of Amenemhat III and Khendjer, only loose blocks survive and for both it was possible to ascertain that their slope was between 54° and 56°. These

² Stadelmann, MDAIK 39, 236.

⁴ Cf. R. Gillings, Mathematics in the Time of the Pharaohs (New York, 1972), 185-7, 212.

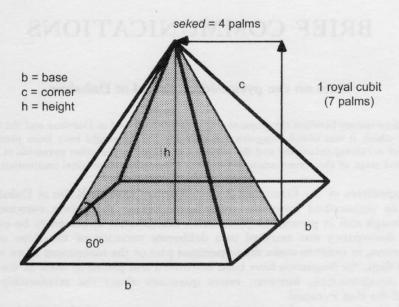
⁶ R. Stadelmann, Die ägyptischen Pyramiden (Mainz, 1985), 101.

¹ R. Stadelmann, 'Die Pyramide des Snofru in Dahschur. Zweiter Bericht über die Ausgrabungen an der nördlichen Steinpyramide', *MDAIK* 39 (1983), 235–6. The location and condition of the pyramidion led Krauss to suggest that the stone was never placed on the top of the Red Pyramid and that, therefore, it did not break during the fall but was smashed on the ground (R. Krauss, 'Zur Berechnung der Bauzeit an Snofrus Roter Pyramide', *ZÄS* 125 (1998), 30).

³ About 187 cm for Amenemhat III (JE 35133 and 35745) and about 141 cm for Khendjer (JE 53045) (measurements taken in the Cairo Museum).

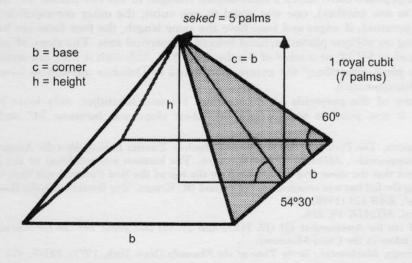
⁵ This triangle is proportional to half the vertical section of the pyramid, i.e. it is a small version (only one cubit high) of half the vertical section.

⁷ D. Arnold, Der Pyramidenbezirk des Königs Amenemhet III. in Dahschur (Mainz, 1987), 13; G. Jéquier, Deux pyramides du Moyen Empire (Cairo, 1933), 30. Jéquier's drawings of the pyramidion of Khendjer might be misleading, since he drew the plain faces, not their projection: the triangle in his drawing is, therefore, the form of the faces of the pyramid (four equilateral triangles with a slope of 60°), not of its vertical section (the seked of which is 5 palms, corresponding to 54°30'. See fig. 1).



Vertical section = equilateral triangle
Slope of the pyramid = 60° (seked = 4 palms)
Shape of Project 1 of the Bent Pyramid

FIG. 1a. Seked of 4 palms and corresponding slope in degrees.



Face = equilateral triangle
Slope of the pyramid = 54°30' (seked = 5 palms)
Shape of Project 2 of the Bent Pyramid and
of the Dahshur pyramidion

FIG. 1b. Seked of 5 palms and corresponding slope in degrees.

measurements are indeed very close to those for the associated pyramidia. In the case of the pyramidion found at Dahshur, however, the discrepancy between pyramid and pyramidion is striking.

According to Perring,⁸ who cleared the corners, the slope of the Red Pyramid was about 43°36'. Petrie could not clear the base but calculated the slope from the rough surface of the core masonry as about 44°36', 'clearly not 45°. The result of a more recent measurement by Polz¹⁰ provided a value of just 45°. Thus, the slope of the pyramid has been given values between 43° and 45°, while the pyramidion has, beyond any doubt, a slope of about 54°30'. Although the idea of a voluntary or involuntary variation of the slope during construction cannot be ruled out,¹¹ it seems that a variation of about 10° would have been very unlikely. Therefore, it is worth considering whether this pyramidion might have been intended for another pyramid in the area.

Amenemhat III chose a slope of 5 palms for his pyramid at Dahshur, and for its inscribed pyramidion now in the Cairo Museum.¹² Amenemhat I also chose the same *seked*,¹³ but his pyramid lies at a significant distance from Dahshur, at el-Lisht. The pyramid of Amenemhat II at Dahshur is badly destroyed and nothing can be said about its slope.¹⁴ Therefore, the possibility that he also used the same *seked* as his predecessor and his successor, that is, 5 palms, can be neither ruled out nor confirmed.

Middle Kingdom kings seem to have preferred inscribed and decorated pyramidia, as the remains of the completed monuments of Senusret II,¹⁵ Amenemhat III (at Dahshur) and Khendjer¹⁶ suggest. Moreover, the surviving Middle Kingdom pyramidia are made of dark stones: black granite for those of Senusret II, Khendjer and the two unfinished pyramidia from Saqqara,¹⁷ dark grey granite for those of Amenemhat III and Merneferra-Ay,¹⁸ and basalt for the pyramidion from Ezbet Rushdi.¹⁹

The pyramidion found at Dahshur, therefore, being undecorated and made of limestone, does not share what seem to be typical Middle Kingdom features. On the other hand, it is difficult to establish common characteristics for Old Kingdom pyramidia, since the surviving examples are limited to the undecorated limestone pyramidia of Khufu's satellite pyramid²⁰ and that of GIIIa (a queen of Menkaura),²¹ and to fragments of the black granite pyramidion of Queen Khentkawes.²² As for the basalt pyramidion found at Abusir in the area of the two pyramids

⁸ I. Perring, in H. Vyse, Operations Carried on at the Pyramids of Gizeh, III (London, 1842), 63-5.

⁹ W. M. F. Petrie, A Season in Egypt (London, 1887), 27.

¹⁰ Stadelmann, MDAIK 39, 235.

¹¹ Although a number of pyramidia have been found, for only the Middle Kingdom pyramids of Amenemhet III (at Dahshur) and Khendjer can the slope of the pyramidia be compared with the slope of the surviving casing of the pyramids.

¹² Now JE 35133 and 35745. See G. Maspero, 'Sur le pyramidion d'Amenemhait III à Dachour', ASAE 3 (1902), 206–8; H. Schäfer, 'Die Spitze der Pyramide Königs Amenemhat III', ZÄS 41 (1904), 84–5; L. Habachi, 'Two Pyramidions of the XIIIth Dynasty from Ezbet Rushdi el-Kebira (Khata'na)', ASAE 52 (1954), 471–9; Arnold, Amenemhet III, 14.

¹³ Stadelmann, Pyramiden, 230-1.

¹⁴ J. de Morgan, Fouilles à Dahchour en 1894-1895 (Vienna, 1903), 30.

¹⁵ W. M. F. Petrie, Lahun, II (ERA 27; London, 1923), pl. xxiv.

¹⁶ Jéquier, Deux pyramides, 19-26.

¹⁷ Ibid. 58.

¹⁸ Habachi, ASAE 52, 472.

¹⁹ Ibid. 475.

²⁰ M. Lehner, The Complete Pyramids (London, 1997), 222-3.

²¹ P. Jánosi, 'Das Pyramidion der Pyramide G III-a', Studia Aegyptiaca 14 (1992), 306-14.

²² M. Verner, 'Excavations at Abusir. Season 1978/1979—Preliminary Report', ZAS 107 (1980), 158. Of the pyramids of Khafra and Udjebten, only the bases of the pyramidia are preserved. For Khafra: LD I, 27; for Udjebten: G. Jéquier, 'Rapport préliminaire sur les fouilles exécutées en 1925–1926 dans la partie méridionale de la nécropole Memphite—La pyramide de la reine Oudjebten', ASAE 26 (1926), 48–9, and La pyramide d'Oudjebten (Cairo, 1928), 3–5.

known as Lepsius 24 and 25 (possibly built for two queens of Neuserra),²³ it may be interesting to note that it bears a strong similarity to the pyramidion found at Ezbet Rushdi, attributed to the Thirteenth Dynasty: both are made of basalt and both were probably meant to be covered by a metal cap. Moreover, the length of their bases is very similar (44 cm, about 6 palms, for the first and 53 cm, about 7 palms, i.e. 1 cubit, for the second) and their slope is the same, about 60°.²⁴ Lepsius interpreted two mounds close to the edge of the cultivation at Abusir as pyramids, which he called 16 and 28,²⁵ and Dodson has suggested that these might be Thirteenth Dynasty monuments.²⁶ So far, however, the excavations of the Czech Institute at the site have not revealed any evidence to support this idea.²⁷

The pyramidion found at Dahshur, then, appears to be similar to the two surviving Old Kingdom limestone pyramidia. A possible answer to the question of its origin may, in fact, lie in the sequence of pyramids built by Snefru. At the end of his reign, Snefru left three completed pyramids, the relative chronology of which has been the subject of several studies. Following Stadelmann's hypothesis, the Bent Pyramid was the first pyramid intended as a true pyramid (although not necessarily Snefru's first pyramid). It was followed by the Red Pyramid, during the construction of which the step pyramid of Meydum was also cased. The final form of the Bent Pyramid is the result of three changes of slope (which will be called Projects 1, 2 and 3), caused by structural problems, rather than by an original intentional design.

With Project 1, the vertical section of the pyramid was meant to be an equilateral triangle: the *seked* was 4 palms, corresponding to a slope of 60° (cf. fig. 1a). Evidence of this first construction can be found in the West and North Descending Corridors at the points where the new portion of corridor joins the original.²⁹ This first project reached at least the height of the West Corridor, but was then abandoned because of a settling in the masonry. This was probably due to subsidence in the foundation rock.³⁰ The base of the pyramid was then enlarged and the *seked* was changed from 4 to 5 palms, corresponding to a reduced slope of about 54°30' (Project 2, cf. fig. 1b). The conditions of the structure, nevertheless, continued to deteriorate to the point when, the pyramid having reached the height of 90 cubits, a radical decision was taken: in order to reduce the weight on the already damaged core, a second change in the slope was necessary and the pyramid was completed using a slope of about 43°–45° (Project 3).

The slope of the pyramidion found at Dahshur, in fact, corresponds to the *seked* chosen for Project 2. The slope of pyramids was established before the beginning of the work, and the pyramidion was probably completed in advance in order to act as a guide for the final smoothing of the faces. When construction was completed, it could then be placed on the top of the monument. This pyramidion, therefore, could have been prepared during the second stage of the construction of the Bent Pyramid, and then abandoned and thrown away when the last variation of the project made it useless.

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²³ M. Verner, 'Abusir Pyramids: Lepsius no. XXIV and no. XXV', in C. Berger, G. Clerc and N. Grimal (eds), *Hommages à Jean Leclant* (BdÉ 106/1; Cairo, 1994), I, 371-8.

²⁴ According to Habachi's drawing (ASAE 52, pl. 18), the pyramidion found at Ezbet Rushdi seems to have a slope of about 70°. According to the dimensions written in the text, however (side-length of 53 cm and corner of 70 cm; ibid. 475–6), the slope can be calculated as about 60°.

²⁵ LD I, pl. 32

²⁶ A. Dodson, 'Two Thirteenth Dynasty Pyramids at Abusir?', Varia Aegyptiaca 3 (1987), 231–2.

²⁶ M. Verner, personal communication.

²⁸ See especially K. Mendelsohn, *The Riddle of Pyramids* (Cambridge, 1974), 88, 114, and R. Stadelmann, 'Snofru und die Pyramiden von Meidum und Dahschur', *MDAIK* 36 (1980), 437–49.

²⁹ J. Dorner, 'Form und Ausmaße der Knickpyramide', MDAIK 42 (1986), fig. 4.

³⁰ V. Maragioglio and C. Rinaldi, *L'architettura delle Piramidi Memfite*, III (Rapallo, 1964), 58–62; D. Arnold, *Building in Egypt* (New York, 1991), 110, 234–5, 238–40.