Elementary District Metered Areas design of looped water distribution networks with multiple sources

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Abstract: the identification of district metered areas (DMAs) in existing water distribution networks is a complex problem, whose solution strongly depends on the type of network. The purpose of the present work is to present a new methodology to identify isolated DMAs in water distribution network having a large number of water sources, such as autonomous reservoirs (for example supplied by pumping stations) directly connected to the distribution network. The method proposed is based on graph theory. Initially, the network is subdivided to the least terms into "elementary" districts, areas of influence of each supply source. In a second step, the resulting elementary areas are conveniently united to form possible layouts of DMAs solutions, respecting the criteria of independency, isolation and self-sufficiency. The union process takes into account the size of districts and the overall network resilience. The method is effectively applied to a case of study and the resulting DMA layouts of the network can be used to compare different preliminary solutions of large network partitioning.

Author keywords: water distribution networks; multi-sources; district metered areas; graph theory; isolated districts; elementary districts.

INTRODUCTION

The design of district metering areas in existing water distribution networks (WDNs) represents a complex problem, as often water networks have been designed in successive stages and not conceived with a district metering perspective. However in the last decade this techniques has been used to improve water distribution system management and efficiency.

The redesign consists in partitioning a WDN into permanent areas, called District Metered Areas (DMAs), in which flow quantities entering and leaving are metered. Two categories of district metering design can be defined. The first one aims to partition the network into permanent district metered areas (DMAs), by means of the insertion of boundary valves; the second one is tailored for multiple source networks and consists in dividing the original network into isolated sectors self-sufficient and independent in terms of water supply (isolated DMAs). This latter category, whenever possible, is even more effective in terms of providing water protection from accidental or intentional contaminations (Di Nardo et al. 2015).

The principal aims of superimposing DMAs to a looped or branched system are: to simplify water balance calculation with flow meter strategically placed on every zone, enhancing in this way the control on water losses and thus the usage of water and energy resources; to limit the effect of a potential contamination by partitioning the network into smaller areas; to manage pressure more efficiently on every zone, saving energy and maintenance costs. The continuous monitoring allows an overall more efficient management of the total system behaviour.

Several studies and technical reports containing general guidelines for dividing a water distribution network into district metered areas are available in literature (Farley, 1985; WRc 1994; Butler, 2000; Morrison et al. 2007; Baker, 2009). The essential factors to take into account are: the size limits of the DMAs, recommended generally between 1,000 and 3,000 users, accondingly to WRc guidelines, or between 2,500 and 12,500 inhabitants (Butler 2000); the connectivity properties (as a result fire flow and pipe bursts conditions, but also water age could be affected by DMAs); and pressure constraints at demand nodes. Other factors can be final leakage level target, implementation and maintenance costs.

For a given WDN, a number of possible DMAs layouts exists and the definition of the final solution, in order to not lower the system performance, should rely on a variety of conflicting objectives, such as cost, reliability and water quality. Many are the methodologies proposed in literature with the aim to redesign water supply distribution network and considerable work has been done to improve and automate the design, planning and management of DMAs.

Due to problem complexity, traditional methods were mainly based on a trial and error approach over the network hydraulic and water quality models to design a WDN into sectors, (Charalambous, 2005; Macdonald and Yates, 2005; Rogers, 2005). After an initial division into districts the solution was evaluated by performing hydraulic simulations under different demand scenarios (average demand, maximum demand, fire flow, pipe burst). The final solution was obtained by manually modifying the water network until the design constraints were met, i.e. an acceptable solution found.

Graph theory (Jacobs and Goulter, 1989) has been proved an useful tool for modelling water distribution network, thus also when solving DMA design problems (Ostfeld and Shamir, 1996; Ostfeld, 2005; Tzatchkov et al. 2006; Sempewo et al. 2008, Herrera 2011). By means of graph principles the structure of the network can be simplified and the complexity of the problems is likewise reduced. The properties of connectivity among nodes within a DMA and reachability of nodes from the water supply sources are extremely important when redesign a water distribution network into DMAs.

Considering size constraints and connectivity properties, Ferrari et al. (2013) recently developed an automatic methodology, based on a recursive bisection algorithm, which determines DMA boundaries so that districts are sized between imposed limits, independent one from another and compliant to minimum pressure requirements.

Another aspect to address is the optimal positioning for boundary valves and flowmeter. In this regard, Di Nardo and Di Natale (2010) developed a decision support system for DMA design using some graph theory approach to identify minimum dissipating power paths. By computing a frequency value for every pipe, proportional to the number of times that a given pipe is found in one of the aforementioned paths, flowmeters are candidate on higher frequency pipes, while boundary valves are inserted on lower frequency pipes.

Using similar principles, Alvisi and Franchini (2014) proposed an automatic procedure to generate different districts layouts, for an assigned number of DMAs, i.e. meters to be installed. Firstly, without performing a hydraulic simulation, possible solutions are found with the aid of the Breadth-First Search (BFS) algorithm: network nodes are grouped in sets, according to their weighted distance from a source node, proportional to head loss. Districts are then defined as groups of node sets having total water demand within the recommended limits. Several possible flowmeters and closure valves locations are then compared, now performing a hydraulic simulation. It is selected the layout that maximises the system resilience.

In general, it is preferable to consider more DMA layouts in the way to support a more flexible decision process. The best solution is always identified by a comparison, which depending on the problem goals and contraints can be based for example on network resilience (Prasad et al 2004; Alvisi and Franchini 2014), pressure and flow deficit indices (Giustolisi et al. 2008b).

Whereas several design criteria and constraints are considered, graph theory approach is often coupled with evolutionary algorithms allowing to evaluate several DMA possibilities (Hajebi et al. 2014; De Paola et al. 2014, Giustolisi and Ridolfi 2014). However the needs of planners are sometimes difficult to traduce in mathematical terms and thus optimization methods based on the definition of a precise multi-objective function cannot be easily applied (Di Nardo et al. 2010).

A significant role on the choice of the technique to adopt is played by the network size and its characteristics. In particular whenever the number of sources is elevated it can be preferable to create isolated districts, each fed by a single or a set of indipendent sources.

In this regard, Di Nardo et al (2013) used graph theory to recognize minimum dissipating power connecting each source to nodes, and thus using a GA to swap the nodes and subdivide the network into isolated DMAs. Some other graph theory approaches of WDN-sub-zoning problem, are described and compared by Perelman et al. (2014), among which graph partitioning and community structures. In particular, the community structure concept, which is a property in common to many of complex systems, is used by Diao et al. (2013) to decompose the network into a layout sub-systems which are seen as basis for DMA planning. However not every distribution network has been developed from the union of communities, thus when dealing with very large water distribution networks densely looped and supplied by multiple sources, developing an automatic procedure to decompose the original network into sub-systems with the required properties can require a more generic tecnique.

Sometimes it can be undesiderable to subdivide very large systems into DMAs of recommended size in one single step. Because most of them were originally conceipted as a single unity, and are still working as such. Moreover for networks fed by an elevated number of sources an eventual partitioning of the network can result in a sensible change in water use for the different sources.

To tackle all these aspects, this study exploits the concept of "influence area" of a supply station. This concept, already proposed in the design phase to build up large networks (Swamsee et al. 2008), can be reversed and used to decompose an existing network into isolated subsystems with indipendent input sources.

An automatic procedure is here proposed for suddividing into DMAs a large WDN supplied by several sources. The method is based on graph theory principles and a criterion of prevalent supply contribute by the different sources. In particular the design criteria adopted are: direct access to water sources; to limit the DMA effects on water use, i.e. changes between sources; to respect of minimum pressure on nodes requirement.

After a preliminary analysis and hydraulic simulation, the network is mapped as an oriented graph, using weights proportional to pipe flows, every node of the network is assigned to its most important water source in terms of demand contribute. As a result the original system is subdiveded into as many influence areas as the number of sources feeding the network. Each area representing a portion of the network predominantly supplied by a given source, so that boundaries closure will not deeply affect the original water use of sources.

In particular influence areas can be then considered as isolated districts, when boundaries are closed, or as mere "functional" districts when not confined by a physical border.

Hereafter they will be referred to as “elementary” districts (eDMAs), for they represent the least possible subdivision of the network into independent and isolated areas.

Once identified, elementary districts can be used as bricks with which compose different DMA layouts. The coupling procedure developed is based on resilience maximization. It is fast and flexible procedure, able to adapt to different objectives and to the level of sectorization the water utility is interested to, leaving open the possibility for a progressive sectorization.

Finally the method effectiveness is tested on large WDN, case study derived as a portion of a real distribution system.Some consideration on the possible application of the method are drawn in the conclusion.

METHODOLOGY

The proposed methodology consists of three main steps:

1. Preliminary analysis of the network, in order to identify main supply sources, that are those regularly active at peak demand conditions;
2. Elementary DMAs (eDMAs) creation by means of graph theory based algorithms. A number of district equals to the number of sources is generated: each source will have its own eDMA and each eDMA is supplied only by one source;
3. Progressive eDMAs union according to size constrains and to a criterion of resilience maximization with an automatic iterative procedure. During the union procedure the physical borders of eDMAs are removed to make the fusion itself possible, and they are close again once the optimal union has been selected, from a functional point of view.

Input Data

The proposed methodology requires the following input data:

- The hydraulic model of the water distribution network, nodal base demand, demand patterns;
- Minimum required pressure over nodes, in the way to verify the feasibility of the resulting solutions;
- A criterion to drive the eDMA union process. In this work a maximum resilience rule is selected.

PRELIMINARY ANALYSIS

A preliminary analysis of the original network configuration is carried out to acquire some topological information, among which is of particular interest the number of source nodes (N_S) present in the distribution system, as this information will be essential in next phase when an equal number of elementary DMAs has to be identified.

Subsequently, a complete hydraulic simulation of a demand driven model of the network is performed through EPANET 2.0 (Rossman, 2000) and pipes flows at peak hour are evaluated. Water losses were not explicitly considered but evenly distributed, included in the estimated nodal demands.

Pipes flow data are used to draw two matrices. Adjacency Matrix (AM), containing all the connections between the network nodes, is defined as:

$$\begin{cases} AM(i,j) = AM(j,i) = 1 \text{ if exists a link between nodes } i \text{ and } j \\ AM(i,j) = AM(j,i) = 0 \text{ otherwise} \end{cases} \qquad \text{(Eq. 1)}$$

Flow-weighted adjacency Matrix (FM), which considers the flow directions through the weight w_{ij} defined as:

$$w_{ij} = \frac{1}{|q_{ij}|} \qquad \text{(Eq. 2)}$$

where the discharge flowing from node i to node j is called q_{ij} and it is a positive value if the flow direction is from node *i* to *j,* negative otherwise.

The FM compilation, where FM is not a symmetric matrix, is carried out following the subsequent law:

$$\begin{cases} FM(i,j) = w_{ij} \quad and \quad FM(j,i) = 0 & ,if\ q_{ij} > 0 \\ FM(i,j) = 0 \quad and \quad FM(j,i) = w_{ij} & ,if\ q_{ij} < 0 \\ FM(i,j) = FM(j,i) = 0 & ,if\ q_{ij} = 0 \end{cases} \qquad \text{(Eq. 3)}$$

ELEMENTARY DISTRICTS CREATION

A number N_S of elementary districts is created, equal to the number of sources identified during the preliminary analysis. Extension of these "functional" eDMAs obviously varies in time with water demand. However, in most cases variation is referred to a limited zone along the border in common between different influence areas, where network nodes are supplied quite equally by two or more sources. For this reason in the presented method, a reference demand condition is considered for eDMAs identification, corresponding to the maximum peak demand.

In order to define each eDMA domain, the network is explored using graph theory algorithms and the FM matrix just created, in order to identify which nodes are belonging to which sources. In particular two algorithms will be used, Breadth First Search (BFS) and Dijkstra algorithm.

Firstly, following flows direction from a given source node Sj, the BFS algorithm allows to obtain a nodes list NL of all nodes supplied by the source. A NL is thus created for every source Sj of the network $NL(Sj)$. If a generic node x is a downstream node of source Sj, this means that the water delivered from node x derives totally or partially from that source.

Due to the looped nature of the network, generally few nodes are reached only by a single source, more frequently, a node is reached from several sources. In the former case, the node can be immediately assigned to the eDMA built around its single source; in the latter and more frequent case, instead, the choice of the eDMA assignation is less obvious and it will be carried out with additional steps. Looking at the example in Fig. 1, the node list of sources *S1* and *S2* are *NL(S1) = {a, b, c, e, f, g, h, m, p}* and *NL(S2) = {a, c, d, f, h, p}*.

Similarly also a sources list (*SL*) for each node can be created, i.e. a list of every source a given node depends on. It can be observed that nodes *b*, *e*, *g* and *m* are reached only from *S1*, while no flow contribute from *S2* arrives to them, so they directly belong to the influence area of the former source, hereafter called eDMA-1, see Fig. 1. Similarly node *d* belongs to eDMA-2. For all other nodes, reached by more than one source, such as nodes *{a, c, f, h, p}*, a univocal assignation rule has to be established: a node *x* belongs to eDMA-*j* if the most of water delivered from node *x* derives from source *Sj*. In other words, this means that the principal source of that node *x* is the DMA j^{th}-source. In practice this result can be achieved through two additional steps: Dijkstra's algorithm (Dijkstra, 1959) and main path identification.

Dijkstra’s algorithm is exploited in order to find out the main paths (i.e. the shortest) connecting node *x* to its supplying sources, using the weighted adjacent matrix FM, see different grey shades for main path in the example of Fig. 1. Since *FM* weights are inversely proportional to the pipe flow (see Eq. 2), the shortest path resulting from the algorithm is the one made up of the pipes conveying the largest flow that connect the given node to a source. The main paths connecting a given node to its multiple sources may have a common part, i.e. waters from the sources mix up before reaching the node. This is the case of nodes *{a, c, f, h, p}* in Fig. 1.

In any case to assign a node to one of its supplying sources, it is necessary to compare the discharge contributes coming from the different sources. At every fork on the main paths, automatically tracing backward the flows from the given node, it is followed the direction from which comes the highest flow. In this way the prevalent supplying source is found. For example, considering node a in Fig. 3 there are two main paths, *p1={S1,* b*,* c*,* a*}* and *p2={S2,* d*,* c*,* a*},* and the common path is *{*c*,* a*}*. So in order to discriminate the main source for node *a*, a comparison is made between the flows of the pipes immediately upstream the fork, pipe *bc* for path *p1* and pipe *dc* for path *p2,* and the highest one determines to which source's influence areas assign node a. Since $q_{dc} > q_{bc}$, then $eDMA(a) = S2$. If two directions are equal in flow, then, always on the mains paths, this time the comparison is made between the flows of the pipes immediately upstream the ones previously considered.

This procedure, reported also in the flow chart of Fig. 2, has to be repeated for every node of the network leading to a subdivision into groups of nodes. Groups are referred to a specific source, gathering its node domain. This means that every group forms an eDMA. However a group of nodes has not been addressed by the procedure yet: it is the set of no-demand nodes, connected to the network by non-active pipes. The nodes of this group and other assignation errors will be assigned by the iterative refining procedure exposed in next paragraph.

Iterative Refining Procedure

The last step of the elementary district creation procedure aims to refine the node assignation, in order to consider also those nodes lying on non-active links, i.e. links not conveying flow. In addition, the effective continuity of each elementary district domain has to be verified. It may happen that some nodes belonging to a group are not directly connected each other, but spaced out by node belonging to a different set. After the closure of pipes in order to create the district boundaries, these nodes will result isolated. Therefore, network nodes are categorized into two different sets: biological and foster nodes. To this aim it is useful introducing a definition of “parent” nodes: for every network nodes its “parents” (note they could easily be more than two) are its immediately upstream nodes (considering every upstream flow direction). Furthermore, this analysis is carried out at peak demand conditions, so the flow directions are not changing. In this way it is possible to establish that all nodes having at least one parent belonging to their same eDMA are called biological; vice versa a generic node *x* is considered foster if one or more of the following definition are met:

- If the node *x* has been previously assigned to an eDMA different respect to all of the assignations of its immediately upstream nodes;
- If *x* has not been assigned yet to any particular eDMA;
- If *x* is a node on a non-active pipe and all immediately adjacent nodes has been assigned to eDMAs different than eDMA(*x*).

Considering Fig. 3, node *h* initially assigned to Source *S2* is a foster node because its parents (nodes *f* and *g*) are both belonging to *S1*. In the same way also node *q* is a foster child, since it is a no-demand node and no discharge is flowing in pipe *dq*. So, during *FM* creation, $FM(d,q)=FM(q,d)=0$ and then the procedure stopped without assigning an eDMA to node *q*, because its source list is empty ($SL(q)=\emptyset$). Node *q* is then considered as foster and needs to be assigned to a proper eDMA.

On contrary, node *p* is not considered at first a foster node, even if it is a grey node in the middle of black ones. Accordingly to the definition above-mentioned, there is at least one of its parents (in this case corresponding to node *h*) belonging to its same eDMA. It will be addressed for correction in a subsequent iteration step.

In fact, the refining method proposed is an iterative and recursive procedure that, at every loop, controls all network nodes and if recognizes foster nodes assigns them to their proper eDMA. Since the *FM* matrix contains only the nodes connections where the discharge is not null, the refining procedure employs also the *AM* matrix, which contains all the links present in the network, independently on flows.

The correction rules implemented for foster nodes founds at every iteration are the following:

- If node x is linked to the rest of the network by one or more pipes with $q \neq 0$, node x will be assigned to the same eDMA of the parents from whom link derives the maximum flow (see node *h* in figure 3 and 4);
- If node x is located on a no flow pipe ($q=0$) and all of the nodes connected (by links) to x belong to a unique eDMA, then node x will be assigned to the same eDMA of the connected nodes (this is the case of node q in fig. 3 and 4);
- Otherwise the generic node x cannot be assigned yet, it will be reassigned in the following loops.

During the iterations it may happen that new foster nodes are discovered or already assigned nodes turn foster again. The procedure of searching and refining stops when no more foster nodes are left.

As a result of the assignation procedure every source corresponds a more or less extended influence area. The elementary districts creation is completed by defining the boundary of each district, which means that in the hydraulic model all pipes connecting nodes belonging to different district are closed (see Fig. 5). In the particular case of WDN supplied by pumping stations having modified network asset, the creation of eDMAs could be coupled with a revision of the pump schedule in order to maximize the partitioning benefits also in terms of energy consumption. However elementary areas can be just a necessary but intermediate step of the proposed DMAs procedure, for this reason the feasibility and respect of every given constraint will be fully addressed in the following union procedure.

PROGRESSIVE eDMA UNION

Once the original network is subdivided into eDMAs, it is possible to assemble them as if they were bricks with which build up different macro isolated DMA layouts (mDMAs).

Generally, it is unlikely and inconvenient to have a large number of DMAs. Even if it implies a greater network control, it is unfeasible from an economical and practical point of view since it would require the closure of a large number of pipes and to install valves between DMAs.

Therefore elementary districts in the following steps are conveniently coupled to the respective neighbouring districts, in order to form bigger isolated districts. A complete coupling method requires the evaluation of all possible combinations of joined districts in a set of two, three, four, etc., in order to find out the optimal grouping layout. However, for multisource networks, the number of possible combination reaches very high values even with low numbers of eDMAs and computational times becomes unfeasible. Therefore, it is essential to reduce this number and this is carried out through a progressive union procedure driven by a performance criteria.

In fact, by sectorizing a WDN, the vulnerability of the network increases because independent districts cannot anymore rely on nearby sources to overcome emergency situations, (Grayman et al. 2009). Thus the union procedure should select most performing unions from resilience point of view: the performance criteria is based on Todini's resilience index (Todini, 2000), which is a good measure of system capability to overcome failures and unexpected water demands (i.e. pipe bursts and fire flow demands).

Todini associates the capability of the system, to react overcoming stress conditions, with the energetic redundancy. According to him, the resilience index I_r of a water distribution system, after appropriate substitutions, can be written as:

$$I_r = \frac{\sum_{i=1}^{N_n} d_i (h_i - h_i^*)}{\sum_{k=1}^{N_s} Q_k H_k - \sum_{i=1}^{N_n} d_i h_i^*} \qquad \text{Eq. 4}$$

where d_i is the nodal demand and h_i the hydraulic head at each node i, with N_n the total number of network nodes; Q_k and H_k are the discharges and head at the N_s source nodes; h_i^* is the minimum pressure head.

These are the main steps of the union procedure:

1. at first, the procedure individuates the “small” elementary districts, whose size is below or equal to N_0 (which could be for instance the smallest size of the obtained eDMAs);
2. each small district is tentatively united to its neighbouring ones, by temporary re-opening pipes on common boundaries; each different union possibility is compared to the others by looking at the respective Todini’s index. For each small district, among all the union possibilities, the one which maximize the resilience index and at the same time respects the minimum pressure criterion is selected;
3. the districts selected for union are joined together by definitively opening the boundaries they have in common. These latter and the other unchanged districts form a new DMA layout;
4. the threshold for minimum number of nodes can be further increased by ΔN (a reference value could be obtained as the difference between the largest and smallest size of eDMAs divided by twice the number of internal sources), and the procedure starts back from point *1*, creating a different district layout starting from the one obtained on point *3* during the last cycle.

In the present paper, the size of districts is evaluated accounting the number of customers supplied within the DMA. The district union procedure proposed is also summarized in the flow chart of Fig. 6 and in the drawing of Fig. 7. The algorithm stops either when the original network is sectorized into a satisfactory number of mDMAs or when only 1 mDMA is left, scenario of course coinciding with the original network.

In this work, Scenario-1 is called the first scenario created with the elementary DMA creation procedure (Fig. 7a), which corresponds to the maximum sectorization of the network; scenario-N, the last scenario resulting from the union procedure (Fig. 7f), with only one mDMA which corresponds to the entire network.

OUTPUTS

The proposed procedure allows for: the division of an existing network into elementary districts (eDMAs), minimum size subsets of the network; and the successive creation of different mDMA scenarios by progressively unifying eDMAs. At every step of the method it is verified, performing a hydraulic simulation, that during the whole simulation period each node is associated with a pressure equal to or greater than the minimum required. If this constraint is not met, then the solution is discarded as unfeasible in favour of other eDMAs unions.

Several possible DMA scenarios are achieved for the water distribution network under examination, different for districts number and district size, depending on the imposed size threshold. These layouts should be seen as a preliminary subdivision of the network, leaving to a second phase the definition of optimal boundaries. For every mDMA scenario is always checked that minimum size and internal connectedness requirements are met.

Performance indices (PIs) are used to compare the new layouts of the sectorized water distribution system and the original system. In particular three PIs were computed to assess the alterations in the hydraulic behaviour due to partitioning:

a. Resilience index: Todini (2000) proposed a resilience index to give an idea of the ability of the network to supply water demands even under stress conditions, as associated with energetic surplus in the system which can be beneficially dissipated inside the network in case of mechanical (pipe bursts, pumps out of service, power outages, etc…) or hydraulic failures (unexpected changes in demands or pressure heads).

b. Entropy index: Awumah et al (1991) defined a PI directly related to the loop degree of the system, i.e. multiple flow paths from the water source to the demand nodes. In case of failures, if a certain pipe is interrupted, the nodes can be supplied through another path, allowing the maintenance of the service to the customers. Because of the subdivision into zones, it is important to control that this ability is not sensibly diminished.

c. Hydraulic Performance pressure related (HP): introduced by Coelho in 1997, it is based on the assertion that, for a network to perform well from a hydraulic point of view, the pressure at every supply point must range between a maximum and a minimum requirement. In particular, a network HP is high if nodal pressures is close to the lower pressure bound, as this means that there is less energy waste while meeting the prescribed demands.

These PIs will also help in later decision process to select the final “best” solution of DMA layout among the ones obtained. To be thorough, also water quality and the ability of the resulting system to meet fire flow conditions should be checked. In fact, the creation of DMAs has been proven to affect the capability of meeting fire flow conditions as well as the reliability and the quality of the water delivered.

In general, the “best” solution will be identified only after all relevant levels in decision process have been involved, taking into account for instance also non-functional, such as water quality issues, administrative patrols, homogeneity of user typologies. Nonetheless, the proposed method could be considered a helpful tool at preliminary stage where it is important to analyse and compare different possible DMA solutions.

CASE STUDY

The methodology presented in this work is expressively thought for multi-sources, highly looped water distribution networks. Effectiveness of the method is thus tested with the application to a large WDN case study. It is a portion of a larger real network, in which the number and position of supply sources was slightly modified (Fig. 10).

The total length of the water distribution network is about 506 km, reaching more than 330,000 end users and with total demand coverage. The web-like network follows the city's street layout and make it a highly looped system. The network is supplied by 7 reservoirs, of constant head, sparse around the network, called Res. A, B, ..., G, from which are named the districts. The proposed method has been implemented in MATLAB Release 2010a. The hydraulic analysis has been performed by using the simulation software EPANET (Rossman, 2000). The head losses are evaluated with the Hazen-Williams formula. Other basic characteristics are reported in Table 1.

The minimum number of customers per district $N_{threshold}$, defined in Eq. 5 and initially set equal to N_0, can be considered dependent on the case study considered. For this reason, in the present study, the size threshold has been made dependent on the minimum and maximum eDMA sizes obtained (namely N_{min} and N_{max}, see Eq. 6) and it will be incremented by the quantity ΔN, Eq. 7, in the way that different scenarios of network partitioning are obtained.

$$N_{threshold,i} = N_0 + (i-1) \cdot \Delta N \qquad \text{Eq. 5}$$

$$N_0 = N_{max} - N_{min} \qquad \text{Eq. 6}$$

$$\Delta N = (N_{max} - N_{min})/(2 \cdot N_s) \qquad \text{Eq. 7}$$

where N_s is the number of internal water sources (7 in this study, as the number of reservoirs); $N_{threshold}, N_0, N_{min}\ and\ N_{max}$ are expressed in number of equivalent costumers.

In order to better show how the presented method works, in this application no maximum size has been set, thus the union process will continue until the last DMA scenario coincides with the entire distribution network.

Results

Accordingly to the proposed method, at first isolated elementary districts are identified inside the original water distribution. This preliminary sectorization is achieved exploring the network as a graph, in particular using Breadth First Search and Dijkstra algorithms.

By means of BFS algorithm only those nodes relying on a single water source can be assigned, so at the first step just a preliminary node assignation to one of the 7 sources is achieved: 2,801 nodes, about 54% of the nodes, are assigned. Successively, by means of the second and third steps, main paths are identified (Dijkstra's algorithm) and compared in terms of discharges along them, making thus possible to achieve the assignation of 4,971 nodes, which is more than 95% of the whole network. Finally, the rest of the nodes is assigned by means of the refining procedure, which not only assigns the nodes that were not in the previous steps (i.e. all the nodes relying on non-active links) but also verifies and eventually corrects already assigned nodes, in the way that all the elementary districts consist in an interconnected set of nodes supplied by at least one of the internal sources of the network.

As a result of the creation of elementary districts, the original network is subdivided into 7 eDMAs, corresponding to the 7 reservoirs. This eDMA layout, called Scenario-1, is represented in Fig. 9.

Follows the elementary districts union procedure, which is driven by a progressively increasing supplied-customers threshold. Starting from the eDMAs scenario (Scenario-1), a second layout of districts is obtained by coupling small eDMAs with the neighbouring ones, the initial threshold is $N_{threshold,1} = 24{,}480$ customers/district (Eq. 5, 6). The second layout of districts is thus called Scenario-2, and it is composed only by mDMAs of size major than $N_{threshold,1}$. It is the results of a recursive procedure: every possible union of small districts with adjacent districts is evaluated, and the best union is selected by a comparison, of feasible solutions (i.e. respecting the minimum pressure requirement), based on resilience index (Todini, 2000). Subsequently new mDMAs layouts are obtained by increasing progressively the threshold size, ΔN by ΔN (where $\Delta N = 3{,}587$, Eq. 7). At each cycle, only the unions leading to the highest improvement of the network are actuated, giving birth from Scenario-1 to 6 more district scenarios, represented in Fig. 10. Note that the Scenario-7 corresponds to the original network, and consider that the number of obtained layouts and the number of reservoirs coincides just by chance, and it is not a result sought by the method.

The union process history can be reconstructed following the line paths in Fig. 11. Each grey column represents a scenario, from 1 to 7, corresponding to the network layouts of Fig. 10. For every district (A, B, …, G) is reported, on the left side of the chart, the number of customers in each eDMAs (Scenario-1). The number of DMAs per scenario is equal to the number of horizontal lines crossing each column. In the same way the number of customers per DMA in a scenario can be calculated summing the initial numbers of connections at the beginning of the corresponding line paths.

The resulting characteristics of the DMA layouts are summarised in Table 2. Each row is a scenario, while in columns are reported: the numbers of DMAs for the layouts; three performance indexes, resilience, entropy and hydraulic performance pressure ralated; and lastly there are 7 more columns reporting the daily total water demand supplied by the seven reservoirs.

As expected, resilience index decreases with the number of DMAs. However, with the proposed approach resilience seems to be not so much affected by sectorization. It has to be noted that resilience values are very high in all scenarios, near unity, due to the significant energy level in the original system. Entropy values, which reflect the densely interconnected nature of the network seems to remain stable, despite the reduction of possible flow paths. So does hydraulic performance pressure related.

In conclusion, the results show only slight changes for all the three performance indicators respect to their values in the original network (last row, Scenario-7). This can be explained by three considerations. First, the eDMAs identified with the proposed approach are essentially already present in the original network from a functional point of view as influence areas of supply sources. Second, the number of reservoirs distributed uniformly enough over the network guarantees a high level of energy in normal conditions. Third, the different mDMAs layouts are selected according to maximum resilience criterion and the respect of minimum nodal pressure requirement.

In Table 2, it is also reported the total water volume (expressed in 10^4 m^3/day) supplied by each reservoir which varies in the different considered layouts. While some reservoirs show negligible changes in their daily consumptions, as in the case of reservoir C, it is not so for some others, which undergo a small increment of water demand, especially in Scenario-1. For example, reservoir G undergoes an increment of daily demand about +10% in the more sectorized scenarios (Scenario-1 and 2), which could be considered a feasible change. The storage supplying capacity and the feasibility of these changes should be checked and it plays a role in the successive decision phase over the most convenient scenario to adopt, both considering water availability and water quality of the different sources.

Another important checks to carry on for the obtained DMA layout, in order to test the goodness of each solution, could be to test fire flow conditions and water quality.

The first control aims to test the ability of the different DMAs layouts to respond to fire flow emergency in compliance to the minimum pressure constraint. This test should be carried out at the most critical situation, therefore at maximum daily demand condition, a fire flow has been superimposed. The fire flow demand has been assumed of 15-30-60 lps, where 30 lps is in line with Italians typical design standards. For each district, an interior node particularly disadvantaged for pressure and located on pipe of small diameter (≤150 mm) has been selected. The nodes selected are also located on the new border of each district, as they are most significantly affected by the closure of boundaries between districts. Although there is no guarantee that the selected node would be the most critical, this can be still considered a reasonable and short way to test the ability of the system to respond adequately to a fire condition (Grayman et al. 2009). The fire flow condition was tested by sequentially and separately applying the fire flow demand to the selected nodes, and the result is always positive for all nodes in case of a demand of 15 lps. For the selected nodes, in the original network, the fire flow control was always positive with pressures at all nodes over 20 meters. When a higher fire flow demand has been tested, such as 30 lps and 60 lps the analysis showed that the pass rate decreases respectively to 75% and 33 % of the DMAs. Overall the fire flow control is positive, especially considering that the tested disadvantaged nodes are on DMA boundaries, which have not been optimized yet, being beyond the aims of the presented work, therefore a better arrangement of border can definitively help the respect of fire flow condition also for this few nodes.

The last control over DMA scenarios is water quality, specifically water age, which can be considered as an information of the potential water quality deterioration in the network. It is the parameter controlled to compare the performance of the network in the original asset and in the DMAs layouts. Both water age and demand-weighted average water age are calculated. The results, reported in Table 3, show that there is no significant difference between the average water ages of the whole system in the alternative designs in comparison to the original values. Of course there can be a noticeable difference in water age node by node, but this does not affect remarkably the overall behaviour of the network, (Grayman et al. 2009).

CONCLUSION

A new methodology for designing isolated DMAs for highly looped multi-source WDNs is proposed by the authors. The method is based on graph theory: it applies graph theory principles and algorithms to determine the least isolated districts subdivision of the existing network. These districts are called “elementary” districts, eDMAs, and should be seen as influence areas of each water source present in the distribution network.

Subsequently the eDMAs are progressively united each other by a union procedure, driven by a maximum resilience criteria and accordingly to their size, obtaining thus different isolated macro-DMA scenario levels of network sectorization. ~~The connectivity properties of the district nodes with the water source are considered in the process of creating the eDMAs, thus also subsequent mDMAs scenarios~~.

The proposed methodology is specifically tailored for large distribution network supplied by a high number of sources, a complex case not usually treated in literature. Here it is hard to decide a priori a number of districts in which subdivide the network, because of its high redundancy and the distribution of the internal supply stations. For this complex systems the method presented is able to identify the influence areas of each internal source and then building different levels of DMAs scenarios without changing the natural dependence of nodes on supply stations. The achieved layouts are all made by self-sufficient and isolated DMAs, without a sensible decrease of the overall performance of the network, in terms of resilience.

Every scenario, every union decision, is obtained by performing hydraulic simulation in order to verify the compliance of the minimum pressure requirements. Moreover, being originated from influence areas, each DMA scenario does not affect significantly the previous water uses coming from the different sources. The resulting sectorization solutions of the network enable the water utility to progressively subdivide a large distribution system into sub-zones, without decreasing the overall performance of the network or altering the sources milking, but obtaining in this way a more reliable and controllable system, possibly enhancing the quality of delivered water and a reducing the risk of contaminant spread.

The methodology developed is applied to a case study in order to test its applicability and effectiveness. Results highlighted how the methodology successfully provides for the division of the water distribution network into a number of elementary districts, equal to the number of sources present in the system, that are characterized by the desired properties: connected with at least a water source, hydraulic independence from each other (i.e., no flow paths are available between two districts), and hydraulic feasibility, meaning that pressure requirements are satisfied.

The procedure is fast for computational time, even when elevated number of sources is elevated. Especially the union of districts phase is fast and flexible and can be potentially be used in real time application when dynamic districts are considered.

Further improvements of the methodology could include district boundaries optimization, considering as selection factors of the best layout also functional and non-functional constraints, such as water quality issues, administrative patrols, homogeneity of user typologies. On a different perspective, elementary districts, being from the functional point of view nothing but influence areas of sources, they could be exploited to address energy optimization problems.

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